國立清華大學命題紙

九十 學年度第一學期 電機工程 學系 博士班研究生資格考試

科目 电磁理論 科號 共 2 頁第 1 頁 *請在試卷(答案卷)內作答

1. The ω-β curve for a certain dispersive channel is such that at frequency $\omega = \omega_0$, the group velocity and phase velocities are equal, and the group velocity is proportional to ω in a certain frequency interval around $\omega = \omega_0$. Find the ratio of group velocity to phase velocity (v_g/v_p) in that frequency interval. (10%)
2. Show that for a hollow waveguide of square cross section ($a = b$), the number of propagating modes at frequency f that is much much larger than cutoff frequency of the dominant mode is approximately equal to $2\pi\mu\varepsilon f^2 a^2$. (10%)
3. The region $z < 0$ is a perfect dielectric, whereas the region $z > 0$ is a perfect conductor. For a uniform plane wave having the electric and magnetic fiels as:

$$\mathbf{E}_i = \mathbf{a}_x E_0 \cos(\omega t - \beta z)$$

$$\mathbf{H}_i = \mathbf{a}_y \frac{E_0}{\eta} \cos(\omega t - \beta z)$$

where $\beta = \omega\sqrt{\mu\varepsilon}$ and $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

Obtain the expressions for the electric and magnetic fields of the reflected wave and the expressions for the total electric and magnetic fields in the dielectric region. Find the surface current density on the surface of the perfect conductor. (15%)

4. Consider a point charge q located at distance of y relative to the origin of a grounded conducting sphere of radius a.
   (i) Please calculate the position of the image charge. (5%)
   (ii) Please calculate the total induced charge on the grounded sphere. (4%)
   (iii) Please briefly describe the behavior of the above induced charge when the point charge moving toward the origin. (3%)

5. Please prove that TEM fields are not allowed in hollow metallic waveguides. (8%)

6. An electromagnetic wave is injected into a hollow metallic waveguide, but its frequency is below the cut-off frequency of allowed modes.
   (i) Please discuss the field amplitude along the waveguide direction. (3%)
   (ii) Please discuss how to experimentally measure the above result. (3%)
   (iii) Please discuss possible negative effects to facilities while under such a low-frequency operation. (2%)

7. Metallic waveguides are used to guide microwaves but not electromagnetic waves of several tens MHz or hundred MHz. Please explain the reason. Please also CLEARLY discuss the very mechanism causing temperature heating in operating metallic waveguides concerned. (6%)

8. The Lorenz gauge relating the magnetic vector potential and the electric scalar potential was first presented by Lorenz in 1867 and plays an important role in electromagnetics. Write down the retarded potentials and the Lorenz gauge in time domain. (11%)

9. The electric scalar potential Φ and the magnetic vector potential $\mathbf{A}$ can be used to express electric and magnetic fields. Write down the field components E_x and B_x in terms of potential Φ and the Cartesian components of potential $\mathbf{A}$. Note that each of the components of potentials appears once and only once in the expressions. (10%)

10. Write down Ampere's law and Faraday's law in time domain in free space in terms of fields $\mathbf{E}$ and $\mathbf{B}$. From these two fundamental laws, show that the wave equation in electric field can be given as

$$\nabla^2\mathbf{E}(\mathbf{r},t) - c_1\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) = c_2\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r},t) + c_3\nabla\rho_n(\mathbf{r},t),$$

and find constants c_1, c_2, and c_3. (10%)