國立清華大學命題紙

九十二 學年度第二學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號 共 頁第 1 頁 *請在試卷(答案卷)內作答

For Problems 1 and 2, logical reasoning and the thinking path leading to your answer are more important than the answer itself. Full or partial credits will be given to those who give good and logical reasoning to the answer.

1. A uniform monochromatic electromagnetic plane wave of angular frequency ω and wavelength λ has an electric field vector expressed by $E = E_0 \cos(\omega t - kz + \phi)\hat{a}_x$, where E_0 is the wave's electric-field amplitude, $k \equiv 2\pi/\lambda$, ϕ is an arbitrary phase, and $\hat{a}_x$ is an unit vector in the x direction.

a. What is the phase velocity of this wave in the $\frac{1}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)$ direction? 5%

b. Does it make sense to define a group velocity of this wave along the $\frac{1}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)$ direction? If it does, derive the expression of the group velocity; if it does not, explain. 5%

2. A cylindrical coordinate system is defined under the three coordinate variables r, ϕ, z. A uniform monochromatic electromagnetic cylindrical wave in vacuum has an angular frequency ω, and a wave vector $\vec{k} = \frac{2\pi}{\lambda}\hat{a}_r$, where λ is the wavelength and $\hat{a}_r$ is the unit vector in the r direction.

a. Find an expression of the electric-field-intensity vector of the wave at the position (r, ϕ, z) and at time t. The electric field is polarized in the z direction. In your answer, clearly show the position dependence of the field amplitude. 5%

b. Find an expression of the magnetic-field-intensity vector of the wave, including the magnitude and direction at the position (r, ϕ, z) and at time t. 5%

c. Find an expression of the time-averaged Poynting vector of the wave at the position (r, ϕ, z). 5%

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3.(a) Write down the basic differential equations which determine the electrostatic field $\boldsymbol{E}$. 3%

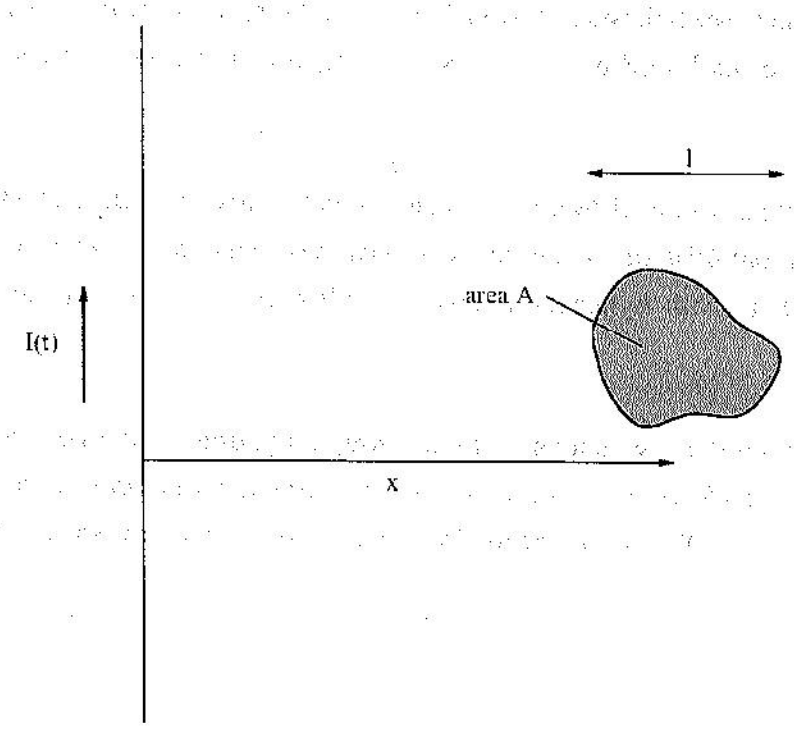
(b) Write down an expression for the *energy density* stored in an electrostatic field $\boldsymbol{E}$. 3%

(c) A hollow conducting sphere of radius $\boldsymbol{R}$ has a charge $\boldsymbol{Q}$ placed on its surface. What is the electric field *inside* the sphere? The potential ? 4 %

(d) What is the electrostatic potential of a point dipole of dipole moment $\boldsymbol{p}$ (in three dimensions)? What is the electric field? 6 %

(e) A point charge of charge $q > 0$ is outside a conducting sphere which carries a total charge $Q > 0$. Is the force between the point charge and the sphere attractive, repulsive, or both? Explain. 4%

4. A planar wire loop of *arbitrary* shape is coplanar with a long, straight wire which carries a current $\boldsymbol{I(t)}$. The loop has a resistance $\boldsymbol{R}$, encloses an area $\boldsymbol{A}$, and is a fixed distance $\boldsymbol{x}$ away from the straight wire. **Assume that x is much larger than the characteristic size l of the loop, and assume that the self-inductance of the loop is negligible.**



(a) Determine the sense of the current induced in the loop (clockwise or counterclockwise) and the direction of the force on the loop (left or right) when (i) $I > 0$ and $dI/dt > 0$ (ii) $I > 0$ and $dI/dt < 0$, (iii) $I < 0$ and $dI/dt > 0$, (iv) $I < 0$ and $dI/dt < 0$. 4%

(b) Find the flux through the loop due to the field produced by the straight wire, and therefore find the mutual inductance M. 3%

(c) What is the current I_{loop} induced in the loop? 3%

(d) What is the magnetic dipole moment of the loop? 4%

(e) Assumeing that $I(t) = I_0 \cos \omega t$, find the force on the loop as a function of x, A, I_0, ω, R, and any physical constants. Find the time average of this force. 5%

(f) Finally, suppose that the self-inductance L of the loop is not negligible. Calculate the time-averaged force acting on the ring. [Hint: write down a differential equation for the current I_{loop} which includes both the self-inductance and the resistance. For the case in which $I(t) = I_0 \cos \omega t$, show that the steady-state solution of this equation is $I_{loop}(t) = \tilde{I} \sin(\omega t - \phi)$, and find $\tilde{I}$ and ϕ.] 8%

5. Derive the reflection coefficient and transmission coefficient for a plane wave incident normally on a plane boundary between two media with the intrinsic impedances η_1 and η_2, respectively. (Use electromagnetics theory) 20 %

6. The wave equation governing the propagation of a plane wave through a lossy medium in the z direction can be expressed as $d^2E/dz^2 = \gamma^2 E$. Write the instantaneous expression for the electric field of the wave. 8%