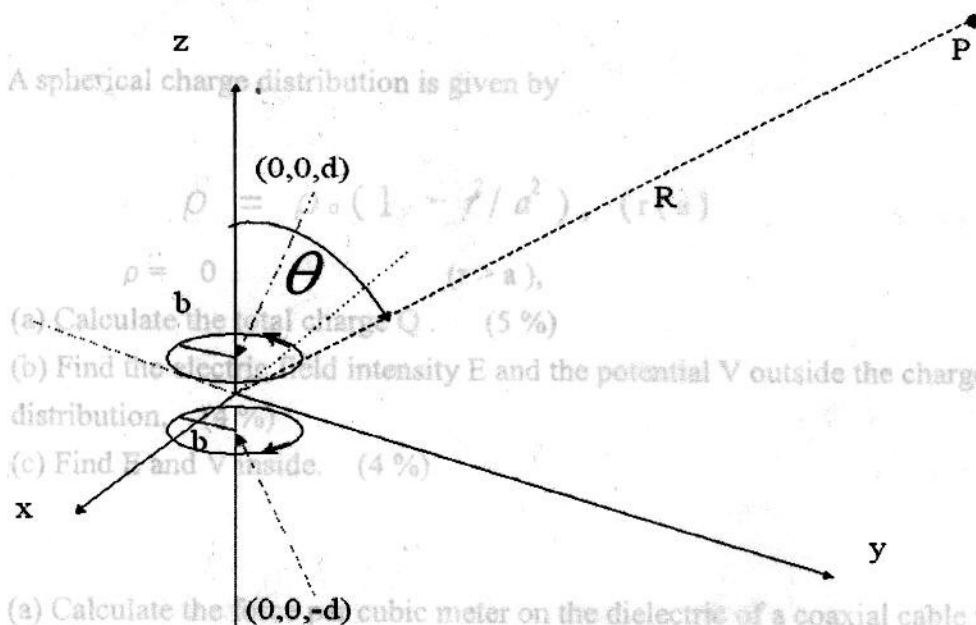


國立清華大學命題紙

九十二 學年度第一學期 光電工程研究所 博士班研究生資格考試
 科目 電磁理論 科號 _____ 共 3 頁第 1 頁 * 請在試卷(答案卷)內作答

- (1) As shown below, find the vector magnetic potential and the magnetic flux density at a distant point of two small loops of radius b separated by $2d$ that carry opposite current I (assuming both b and d are much less than the distance R).
 (15%) *lowest-order propagating mode (11 %)*



- (2) Consider a hollow metal waveguide that is filled with a medium characterized by $\epsilon_r = 2.25$ and $\mu_r = 1$. A kind of propagating modes in the waveguide is described as $E_z(x,y,z) = E_{z0}(x,y) \exp(-\gamma z)$, where $E_{z0}(x,y)$ represents the field distribution in the transverse direction, and γ is defined as $\gamma = j[\omega^2 \mu \epsilon - (m\pi/a)^2 - (n\pi/b)^2]^{1/2}$ with the following definitions:

ω : equal to $2\pi f$

m and n : can be arbitrary positive integers

a and b : fixed positive constants

$$j = (-1)^{1/2}$$

Answer the following questions.

- (a) What is the cutoff frequency of the lowest-order propagating mode considered here? (4%)
- (b) Draw the dispersion diagram (i.e., the ω - β diagram) for the lowest-order

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 科目 電磁理論 科號 共 3 頁第 2 頁 *請在試卷(答案卷)內作答

propagating mode considered here. (6 %)

- (c) Now, assume the filling medium is lossy that $\epsilon_r = 2.25 - j 0.1$ (a complex value). Compute the energy loss per unit propagation distance for the lowest-order propagating mode. (11 %)

- (3) A spherical charge distribution is given by

$$\rho = \rho_0 (1 - r^2/a^2), \quad (r < a)$$

$$\rho = 0 \quad (r > a),$$

- (a) Calculate the total charge Q . (5 %)
- (b) Find the electric field intensity E and the potential V outside the charge distribution. (4 %)
- (c) Find E and V inside. (4 %)
- (4) (a) Calculate the force per cubic meter on the dielectric of a coaxial cable whose inner conductor has a radius of 1 mm and whose outer conductor has an inner radius of 5 mm. The dielectric has a relative permittivity of 2.5. The outer conductor is grounded, and the inner conductor is maintained at 25 kilovolts. (12 %)
- (b) Show that the electric force near the inner conductor is about 300 times larger than the gravitational force if the dielectric has the density of water, namely 103 kilograms/meter³. (5 %)

(7) A laser beam propagating in the z direction has a finite beam diameter in the $x-y$ plane. For the following three questions, no credit will be given without physical and logical explanations. In your answer, you must use the phrase "Because....., therefore.....". Formulas are not very helpful in answering the questions.

- (5) (5 %) Argue that a laser beam of a finite diameter must diffract.
- (5) Two semi-infinite large homogeneous materials are separated at $z = 0$. The material on the left has a positive refractive index and that on the right has a negative one. A plane wave is incident on the interface with an incident angle of θ from left to right. Discuss the reflection angle (direction) in the first material and

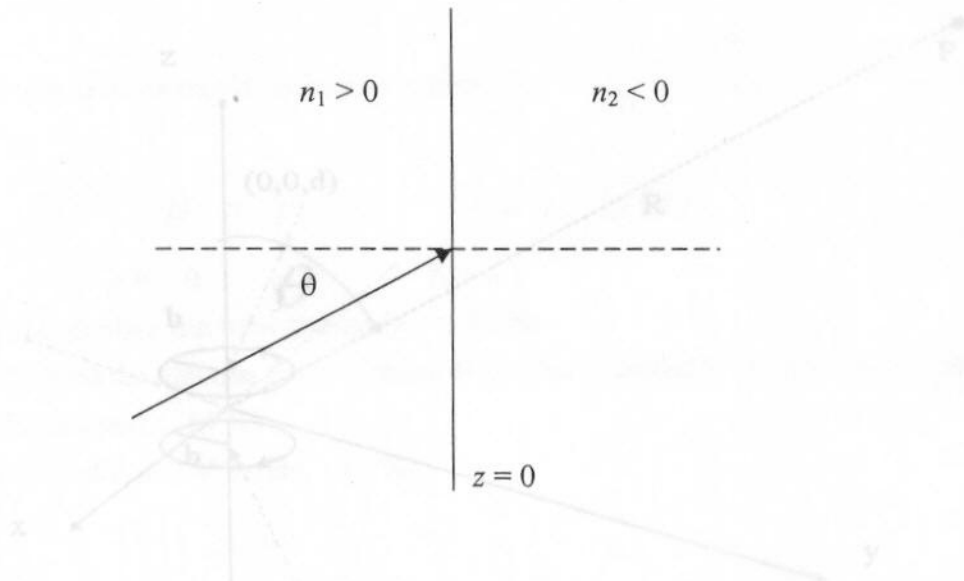
國立清華大學命題紙

九十二 學年度第一學期 光電工程研究所

博士班研究生資格考試

科目 電磁理論 科號 共 3 頁第 3 頁 *請在試卷(答案卷)內作答

- (1) As shown below, find the vector magnetic potential and the magnetic flux density the refraction angle (direction) in the second material for the cases of $|n_1| > |n_2|$ and $|n_1| < |n_2|$. (12 %)



- (6) a. (6 %) Plot an ω - β diagram in which the group velocity of an electromagnetic wave is larger than its phase velocity. In what situation would an electromagnetic wave's group velocity larger than its phase velocity?
- b. (6%) Plot an ω - β diagram in which the group velocity of an electromagnetic wave is in the opposite direction of its phase velocity. In what situation would an electromagnetic wave's group velocity is in the opposite direction of its phase velocity?
- (7) A laser beam propagating in the z direction has a finite beam diameter in the x - y plane.
- a. (5%) From Maxwell's equations, prove that there must be z component electric fields associated with the beam.
- b. (5 %) Argue that a laser beam of a finite diameter must diffract.
- Answer the following questions.
- (a) What is the cutoff frequency of the lowest-order propagating mode considered here? (4%)
- (b) Draw the dispersion diagram (i.e., the ω - β diagram) for the lowest-order