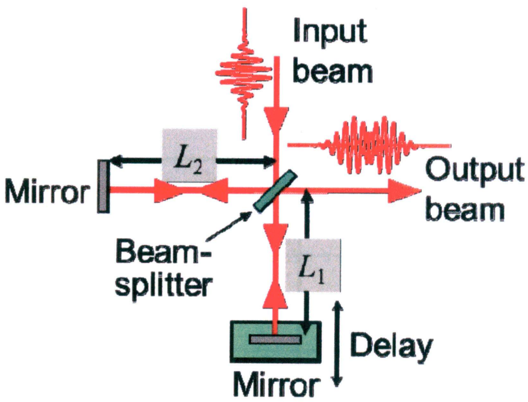
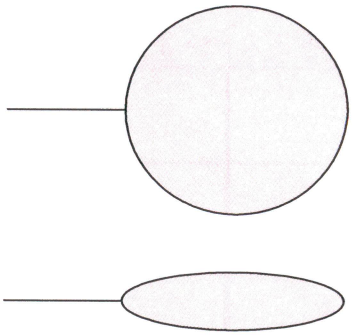
國 立 清 華 大 學 命 題 紙

1. (10%) Show the step-by-step derivations for the ABCD matrix of a thin lens.

2. (15%) Interferometers are often used in displacement detection.
   (a) What kind of interferometer is shown in the following figure on the left? (2%)
   (b) If the input beam has a wavelength of $\lambda_0 = 1$ μm, how many fringes will you observe in the output if the distance between the beamsplitter and the mirror with variable delay (L_1) is increased by 10 um? (2%)
   (c) What are the conditions to have the clearest contrast of the fringes in the output? (2%)
   (d) What do you expect to see in the output if the input beam is a white light? (2%)
   (e) What do you expect to observe on a film made of liquid dishwashing soap? What's the difference when it is placed vertically and horizontally? Please draw and explain. (7%)





3. (10%) (a) The intensity of the Gaussian beam can be expressed as $I(r,z) = I_0(z)e^{\frac{-2r^2}{w^2(z)}}$, where $I_0(z)$ is the peak intensity at the center of the beam spot. Please express the power P of this Gaussian beam as a function of the peak intensity $I_0(z)$ and its spot size $w(z)$. Use your expression, estimate the intensity of the beam at z=0. (6%) (b) What fraction of total power is contained in a hole of diameter $2w(z)$? (4%)

4. (15%) (a) Given an arbitrary 2D E-field distribution f(x,y) at the entrance (z = 0), how to calculate the E-field distribution g(x,y) at a distance z = L (if 0 < z < L is vacuum) in terms of "Fourier optics" perspective? (10%) (Hint: No need to deduce the exact formula. Description of the working principle is sufficient.) (b) The beam quality can be typically improved by a "spatial filter" consisting of a pinhole sandwiched between two lenses. Please explain the working principle of this design in terms of "Fourier optics". (5%)

5. (15%) (a) What is the physical origin of the real part of a refractive index and its influence to the propagation of an electromagnetic wave? (5%) (b) What is the physical origin of the imaginary part of a refractive index and its influence to the propagation of an electromagnetic wave? (5%) (c) What is the physical origin and consequence of material dispersion in optics? (5%) To receive credits, you have to discuss and prove your answers.

6. (10%) An optical isolator is one optical component that just allows light traveling in one direction and blocks it in the opposite direction. Please design one isolator and explain the working principle carefully!!

7. (15%) A ring cavity consists of three mirrors of R1 = 95%, R2 = 93%, and R3 = 90% in free space. To form the ring cavity, the mirrors are arranged with the following inter-mirror spacing: l_{12} = 0.5 m, l_{23} = 0.5 m, and l_{31} = 0.3 m. The only losses of this cavity are those from the transmission of the mirrors.
   (a) What are the round-trip time and the longitudinal mode spacing of this cavity? (5%)
   (b) Find the finesse and the longitudinal mode width of this cavity. (5%)
   (c) What are the cavity decay rate, the photon lifetime, and the Q factor for λ= 1 μm? (5%)

8. (10%) Consider a plane-wave wavepacket containing a single photon traveling in the z direction, with the complex wavefunction:

$$U(r,t) = exp[-\frac{(t-\frac{z}{c})^2}{4\tau^2}]exp[j\omega_0(t-\frac{z}{c})]$$

Find the uncertainties in its time and z direction.