國 立 清 華 大 學 命 題 紙

102學年度第一學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 共 3 頁第 1 頁 ＊請在試卷(答案卷)內作答

1. (15%) A static charge distribution produces a radial electric field

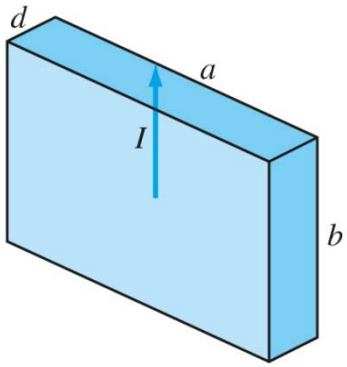
$$E = A\frac{\exp(-br)}{r}e_r$$

where A and b are constants, e_r denotes the unit vector along the radial direction.

(a) (10%) What is the charge density? Sketch it.

(b) (5%) What is the total charge Q?

2. (10%) You are given a thin metal plate of length b, width a, and thickness d (d<<a) that carries a steady current I. Assume you can ignore all the end effects, find the magnetic flux density inside the metal plate.



3. (15%) The E-field of a uniform plane wave propagating in a dielectric medium is given by

$$\vec{E} = 3\cdot\sin\left(10^8\cdot t-\frac{y}{\sqrt{3}}\right)\vec{x}+\cos\left(10^8\cdot t-\frac{y}{\sqrt{3}}\right)\vec{z}\quad [\mathrm{V/m}]$$

(a) (4 %) Determine frequency and wavelength of this EM wave.

(b) (4 %) Describe the polarization of the wave.

(c) (3 %) What is the dielectric constant of the medium?

(d) (4 %) Find the corresponding H-field.

4. (10 %) When there is a relative motion between a time-harmonic source and a receiver, the frequency of the wave detected by the receiver tends to be different from that emitted by the source. This phenomenon is known as the Doppler effect. Let us assume that a light transmitter of a time-harmonic wave of a frequency f moves with a velocity u (assume u << c) at an angle θ relative to the direct line to a stationary receiver.

(a) (4%) Please derive and show that the frequency of the received wave is $f' = \dfrac{f}{1-\frac{u}{c}\cos\theta}$

(b) (3%) If the transmitted signal has a spectral linewidth of $\Delta\upsilon$, what would be the linewidth of the received signal after the Doppler effect?

(c) (3%) How would the linewidth of the received signal change if the target has a rough surface?

5. (15%) Consider the equivalent circuit of a differential segment of transmission line:

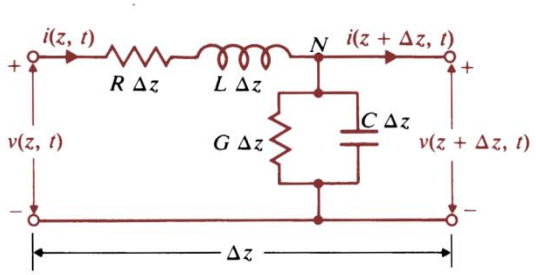


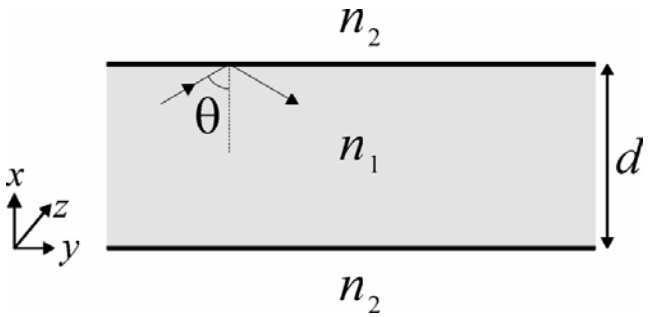
FIGURE 9–5
Equivalent circuit of a differential length Δz of a two-conductor transmission line.

(a) (5%) Use Kirchhoff's voltage law to show that $\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$. (5%)

(b) (5%) Use Kirchhoff's current law to show that $\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$. (5%)

(c) (5%) Show that for a lossless line, the signal speed and impedance are $\frac{1}{\sqrt{LC}}$ and $\sqrt{\frac{L}{C}}$, respectively. (5%)

6. (20%) A symmetric planar waveguide (see figure below) has a core thickness *d* of 3 μm. Ignoring the dispersion of the waveguide material, we find the indices to be n_1(core) = 1.50 and n_2(cladding) = 1.46.



(a) (5%) Is the waveguide single-mode or multimoded at λ = 1.5 and 1.3 μm?

(b) (5%) What is the range of wavelength in which this waveguide is single-mode?

(c) (10%) The guiding mechanism is typically justified by total internal reflection in ray optics, where an incident angle θ (refer to figure) greater than the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$ is required. Does that mean there will be no "cut-off" as long as the optical beam is normally incident (θ = 90°)? Why?

7. (15 %) An inductor is formed by winding N = 10 turns of a thin conducting wire into a circular loop of radius a = 10 cm. The inductor loop is in the x-y plane (surface normal along z) with its center at the origin, as shown below. In the presence of a uniform magnetic flux density

$\vec{B} = 0.2(2\hat{a}_y + 3\hat{a}_z)\sin(10^3 t)$ Tesla, where t is in units of second, find

(a) (5%) the magnetic flux crossing the inductor loop.

(b) (5%) the electromotive force induced in the inductor loop.

(c) (5%) the direction of current flow in the inductor loop when an observer looks into the –z direction. (Give an answer of clockwise or counter clockwise direction and provide your reason to reach the answer.)

(make sure you give the correct units for answers (a) & (b))