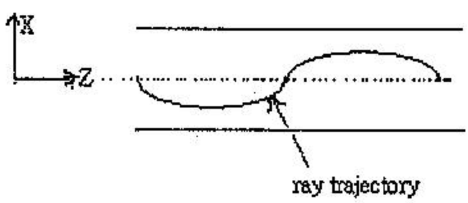
國立清華大學命題紙

九十三學年度第二學期光電工程研究所博士班研究生資格考試

科目　光電子學　共三頁　第一頁　　　　　＊請在試卷（答案卷）內作答

1. Consider the wave propagation in a graded-index medium, as shown below. Also consider the refractive index distribution as a one-dimensional distribution, i.e., assume the distribution to be $n(x) = n_0 \{1 - (x/x_0)^2\}^{0.5}$, where n_0 and x_0 are constants. To find the ray trajectory, one may start with the local slope of the trajectory as
$dx/dz = k_x / k_z$, where k_x and k_z are the x- and z- components of the propagation constant k at position (x,z) in the medium.
   (a) Prove that the slope satisfies
   $$\beta^2(dx/dz)^2 = (k_0^2 n_0^2 - \beta^2) - k_0^2 n_0^2 (x/x_0)^2$$
   where k_0 is the propagation constant in free space and $\beta = k_z$. (12 %)
   (b) Try the solution form $x = C \sin(Dz + E)$ and then solve the equation in part (a). (3 %)



2. When two beams intersect with each other, there would be a fringe generated at the plane parallel to the intersection plane due to interference.
Use mathematics to derive a distribution of the fringe. (10 %)

3. The TEM_{oo} mode of a YAG laser at 1 μm is characterized by a Rayleigh range z_o = 50 mm.
   (1) Find the beam spot size and wave front radius of curvature at the beam waist. (4%)
   (2) Find the beam spot size and wave front radius of curvature at a distance 100 mm away from the beam waist. (4%)
   (3) A focusing lens with f = 100 mm is placed at a distance of 100 mm after the beam waist to reshape the Gaussian beam. Sketch a diagram to show the shape of the input and output beams. Mark the position z' of the output beam waist and find the value of z'. (9%)

4. In a 4-f spatial filtering system, a mask with horizontal slit-aperture as shown in Fig.(a) is placed at the Fourier plane. A square-shaped pattern as shown in Fig.(b) is placed at the object plane.
   (1) Sketch a diagram to show the configuration of optical system. (4%)
   (2) Sketch the resulted pattern on the image plane. Explain your result. (4%)

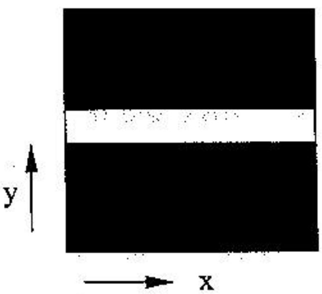


Fig.(a)

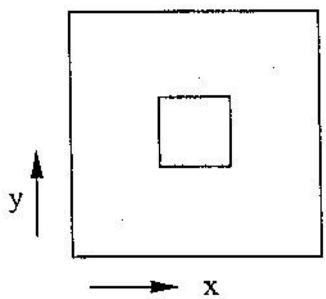


Fig.(b)

5. (a)Write down the Maxwell equations.(b) For source free medium, derive the wave equations for the electric and magnetic fields and show that, in general form, the solutions of the equations representing wave motion. (6%)

6. Show that (a) $e^{j\vec{k}\cdot\vec{r}}$ representing a plane wave (b) e^{jkr}/r representing a spherical wave. (6%)

7. Construct the Jone's matrix of a wave-retarder with fast axis along x direction. By using Jone's calculus, show that a linear polarized light, with polarization 45° to the x axis, turns into a circular polarization when it passes through the quarter wave plate. (6%)

8. For an uniaxial crystal, using the normal surface diagram, show that schematically, the wave front direction is different from the energy direction in general. Along which directions in crystal do the wave front and the energy propagate in the same direction? (7%)

9. Could a concave mirror of a radius curvature = 10 cm and a convex mirror of a radius of curvature = 20 form a stable Fabry-Perot resonator? If it can, determine

the range of separation between the two mirrors for such a stable resonator. If it can't, explain your assertion. (10%)

10. The classic prediction of the energy spectral density of a blackbody radiator was given by $\frac{8\pi\nu^2}{c^3}kT$. Planck overturned the theory and limited the high frequency bandwidth of blackbody radiation. What is the thinking breakthrough that Planck challenged the old theory? In other words, what concept did Planck postulate to create the new correct theory? (5%)

11. (10 %) A typical mercury lamp emits the following colors of light.

(a) Violet-blue color at 365.01 nm, 407.781 nm, 435.835 nm (strong), 491.604 nm (weak)

(b) Green color at 546.074 nm (strong), 576.959 nm (strong)

(c) Yellow color at 579.065 nm (strong)

Estimate the coherence length and coherence time of a typical mercury lamp.