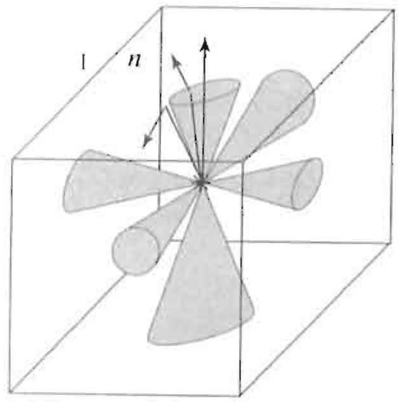
PhD Qualify exam Photonics I 10/21/2014

There are 8 problems in 3 pages with total score of 100

1. (10%)
   - (a) Please explain the condition for total internal reflection at the boundary between two media of different refractive indexes (2%).
   - (b) Assume that light is generated in all directions inside a material of refractive index $n = 3.6$ cut in the shape of a cube shown below. The material is surrounded by air with unity refractive index. What is the angle of the cone of light rays? What happens to the other rays? (4%)
   - (c) Estimate the ratio of the optical power that is possible extracted from the material to the total generated optical power? (4%)



2. (15%)

You are to consider reflection/transmission issue of a TM polarized plane-wave of vacuum wavelength λ between two semi-infinite, isotropic, loss-less media. The relative permittivity and relative permeability are given in the figure for each medium. In the questions below, you should clearly state your reasoning and show all of your works.

(a) (4%) Write out the field expressions for the incident (E_i, H_i), reflected (E_r, H_r) and transmitted (E_t, H_t) waves. You should clearly designate the field orientations using the coordinate given in the figure.

(b) (4%) From your expressions, relate the reflection angle (θ_r) and refraction angle (θ_t) to the incident angle.

(c) (3%) Find out the incident angle so there is no reflection, i.e., $r=E_r/E_i=0$.

Express it in terms of the relative permittivities.

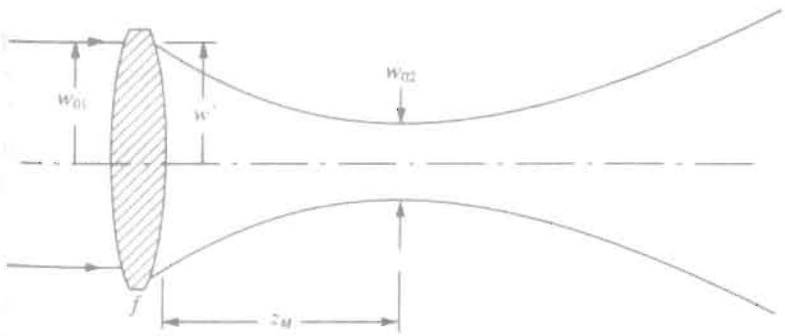
(d) (4%) Using a power approach, express the transmission coefficient t= E_t/E_i in terms of r, incident, refraction angles, and the permittivities.

3. (10%)

As shown in the figure, a thin lens with focal length f = 157 cm is used to focus a large-diameter Gaussian beam with a planar wave front with w_{01} = 1 mm and λ_0 = 1 μm.

(a) Where after the lens will be the focal point (z_M = ?)? What would be the spot size w_{02} and radius of curvature R there? (5%)

(b) Where after the focal point would the laser beam again diverge back to $w = w_{01}$? What would be the radius of curvature R there? (5%)



4. (15%)

(a) (8%)Consider a plane wave with wavelength λ passing through a transmitting grating that has a single spatial frequency ν, under what circumstances the wave will be blocked, i.e. cannot be transmitted? explain your answer.

(b) (7%) What does a lens do in the image processing in terms of spatial frequency manipulation?

5. (15%)

5-1 (8%)The $\omega - \beta$ curve for a dispersive channel is approximated by $\omega = \omega_0 + k\beta^2$ in the vicinity of $\omega = 1.6\omega_0$, where k is a positive constant. Please find (a) the phase velocity for a signal of $\omega = 1.6\omega_0$, and (b) the group velocity for a narrow-band signal having the center angular frequency of $1.6\omega_0$

5-2 (7%) The complex refractive index of a material is given by $N = n + ik$ and the corresponding complex dielectric function is given by $\varepsilon = \varepsilon_1 + i\varepsilon_2$. For strongly absorbing materials ($\varepsilon_2 >> \varepsilon_1$), please show $n \approx k$. (Consider nonmagnetic materials).

6. (10%)

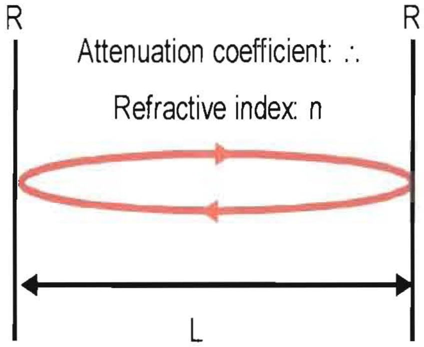
There is a transparent material of thickness 10±0.1 mm (1% uncertainty). Please design a method to experimentally determine whether the material is isotropic or uniaxial.

7. (15%)

Consider electromagnetic waves resonating inside a Fabry-Perot cavity as shown in the figure below. R is the reflectivity at the two end of the cavity. α is the attenuation coefficient and n is the refractive index.

(1) (7%) What is the finesse?

(2) (8%) What is the photon life time?



8. (10%)

Consider a plane-wave wavepacket containing a single photon traveling in the z direction, with the complex wavefunction:

$$U(r,t) = exp[-\frac{(t-\frac{z}{c})^2}{4\tau^2}]exp[j\omega_0(t-\frac{z}{c})]$$

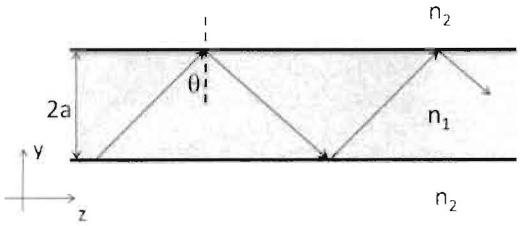
Find the uncertainties in its time and z direction.

PhD Qualify exam Photonics II 10/21/2014

There are 9 problems in 3 pages with total score of 100

1. (10%)

For a symmetric slab waveguide, a slab of width 2a with refractive index n_1 is surrounded by a cladding of smaller refractive index n_2. Assume a monochromatic TEM plane wave bouncing back and forth in the slab. To build up a mode propagating in the slab waveguide, the wave needs to satisfy the self-consistency condition. (a) Explain the self-consistency condition for waveguiding in a dielectric slab waveguide. (b) Assume a phase shift of φ is introduced by each internal reflection at the dielectric boundary. Please derive an expression to represent self-consistency condition in terms of the wavevector (in vacuum) k_0, phase shift φ, slab thickness a, angle θ, n_1 and n_2.



2. (10%)

(a) Suppose the propagation constant of the mp mode, where m,p = 0,1,2... , in a step-index fiber is

$$\beta_{mp} = n_1 k_0 \left[1 - \frac{(m+2p)^2}{M} \Delta \right] \qquad \text{for } V \gg 1 ,$$

where V is the normalized frequency, n_1 is the refractive index of the core, k_0 is the free-space wave number, M is equal to $b \cdot \omega^2$ (b being a constant) and Δ is a constant.

Calculate the group velocity of the mp mode. (6%)

(b) What is the value of normalized frequency for a single-mode step-index fiber ? (4%)

3. (10%)

(a) About laser Spectral line shape, please explain the following items.

(i)Homogeneous broadening and Inhomogeneous broadening (1%)

(ii)Lifetime broadening, Collision broadening, Doppler broadening (1%)

(b) Explain the details of "laser cooling" and its working principle? (4%)

(c) The Nobel Prize in Chemistry 2014 was awarded jointly to Eric Betzig, Stefan W. Hell and William E. Moerner "for the development of super-resolved fluorescence microscopy". What is super-resolved fluorescence microscopy? (4%)

4. (10%)

Please answer the following questions briefly:

(a) How can you transform a laser amplifier into a laser? (2%)

(b) Is it possible to achieve population inversion in a two-level pumping scheme? If yes, how does it work and what is the pumping requirement? If not, why not? (2%)

(c) Please draw the energy levels and all the corresponding routes of decays, pumping, absorption, and emission according to the following rate equations. Assume they are from a three-level pumping scheme. Mark the rate or the time constant for each route. (6%)

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - (N_2 - N_1)W_i$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + (N_2 - N_1)W_i$$

5. (15%)

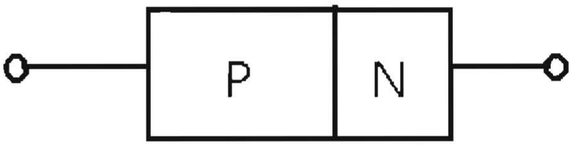
A Fabry-Perot laser, comprising two mirrors separated by a distance d, contains a gain medium with a gain coefficient g. Suppose the mirror reflectances are R_1 and R_2 and light propagate inside the Fabry-Perot with an extra attenuation loss coefficient a. Please derive the threshold gain g_{th} that can be expressed by $g_{th} = a + \frac{1}{2d}\ln\frac{1}{R_1 R_2}$

6. (10%)

(10%) Why is GaAs (a III-V semiconductor) widely used in light-emitting devices while Si (the flagship material of the electronic industry) is rarely seen in light generation? Justify your answer.

7. (10%)

The following represents a pn junction diode structure.



Use this structure to explain the principle of operation of a laser diode.
Is it a forward biased or reverse biased diode? Draw the bandgap diagram when biased.

8. (15%)

Consider an electro-optical modulators:

a. (2%) Provide a schematic sketch of a planar lightwave Mach-Zehnder modulator. Try to be as detailed as you can.
b. (5%) Derive a proper mathematical formula for the output of such modulator when the input may contain electrical ac small signal and dc signal.
c. (8%) Discuss and show how the dc signal could affect your output.

9. (10%)

Explain the optical Kerr effect, in terms of the refractive index, and its relation to the self-phase modulation.

國立清華大學命題紙

一百零三學年度第一學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 共____頁第____頁 ＊請在試卷(答案卷)內作答

1. (15%) Show that in a good conductor the magnetic field of a plane wave lags the electric field by 45°, and find the ratio of their amplitude (intrinsic impedance).

2. (10%) Consider a rectangular waveguide, infinitely long in the z-direction, with a width (x-direction) *2* cm and a height (y-direction) *1* cm. With the wave equation which describes the **E** and **B** fields of the lowest modes, and find the phase velocity and group velocity, accordingly, if the lowest mode can propagate.

3. (10%) Describe and explain in detail the basic working principle of the direct current motor.

4. (10%) In waveguide theory, much effort is dedicated to solving the "modes". Can we analyze the electromagnetic wave propagation in a waveguide without knowing the modes? Justify your answer.

5. (10%) Consider transmission lines:

   (a) (3%) Sketch the lumped-element circuit model for a differential length.

   (b) (5%) From your sketch, derive the generalized transmission-line equations.

   (c) (2%) From your results in (b), derive the expressions for the wave propagation constant for a lossless transmission-line.

6. (5%) Maxwell's equations represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism. Please write down and explain briefly the following four equations and derive the wave equation.

(a) (0.5%) Gauss' law for electricity

(b) (0.5%) Gauss' law for magnetism

(c) (0.5%) Faraday's law of induction

(d) (0.5%) Ampere's law

(e) (3%) Wave equation

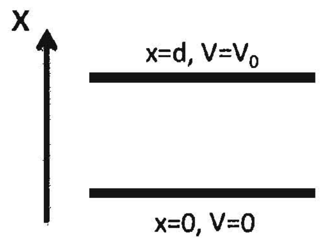
7. (10%) Write down boundary conditions for the Electric field and the Magnetic field (4%) and apply these boundary conditions to examine the behaviour of EM waves at the following three interfaces.

(a) (2%) Dielectric - Dielectric Interface

(b) (2%) Dielectric - Perfect Conductor

(c) (2%) Conductor-Conductor (steady state current)

8. (15%) Consider two infinite, plane, parallel, perfectly conducting plates occupying the planes x=0 and x=d and kept at potentials V=0 and $V=V_0$, respectively, as shown below.



(a) (2%) Does the potential between the region between the plates satisfy Poisson's or Laplace's equation? Please explain.

(b) (5%) Please find the potential distribution in the region between the plates.

(c) (8%) Assume that the region between the plates is filled with two dielectrics having permittivity ε_1 for $0<x<t$ (region 1) and ε_2 for $t<x<d$ (region 2). Please find the solutions for the potential in the two regions $0<x<t$ and $t<x<d$.

9. (15%) When there is a relative motion between a time-harmonic source and a receiver, the frequency of the wave detected by the receiver tends to be different from that emitted by the source. This phenomenon is known as the Doppler effect. Let us assume that a light transmitter of a time-harmonic wave of a frequency f moves with a velocity u (assume $u << c$) at an angle θ relative to the direct line to a stationary receiver.

(a) (5%) Please derive and show that the frequency of the received wave is $f' = \dfrac{f}{1 - \dfrac{u}{c}\cos\theta}$

(b) (5%) If the transmitted signal has a spectral linewidth of $\Delta\upsilon$, what would be the linewidth of the received signal after the Doppler effect?

(c) (5%) If the stationary receiver (target) has a rough surface comparable to the wavelength of the light, how would the linewidth of the received signal change? Why?