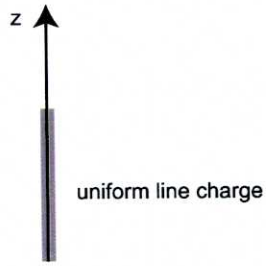
國　立　清　華　大　學　命　題　紙

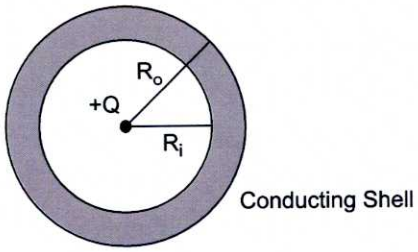
九十五 學年度第二學期 光電工程研究所 博士班研究生資格考試 (95下)

科目 電磁理論 科目代號 共 4 頁第 1 頁 ＊請在試卷【答案卷】內作答

1. (15 %) (a) Gauss's law asserts that the total outward flux of the E-field over any closed surface in free space is equal to the total charge enclosed in the surface divided by ε_0. Use Gaussian law to determine the electric filed intensity of an infinitely long, straight, line charge of a uniform density ρ_l in air. (b) If a line charge with finite length L is considered instead, what is the formula for the electric field intensity along the axis of the line? (hint: for part(b), find the electric potential first)



2. (5 %) (a) A positive point charge Q is at the center of a spherical conducting shell of an inner radius R_i and an outer radius R_o. Please draw E and V as functions of the radial distance R. (b) How about if it is a dielectric shell? (Calculation is not necessary)

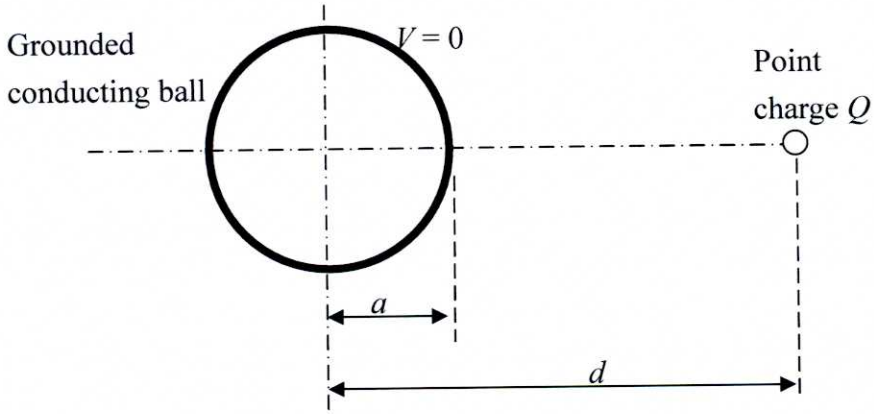


3. (10%) (a) (5%) What is the assumption made in deriving the differential form of Faraday's law of induction $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ from the integral form of Faraday's law $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}$? (b) (5%) If the assumption is removed, how one would modify the expression $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$. Hint: recall the expression for a moving wire in a time-varying magnetic field $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint \vec{u} \times \vec{B} \cdot d\vec{l}$, which can't be reduced to $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ without some assumption. So, what's wrong with the Faraday's law of induction in the four Maxwell's equations?!

4. (10%) A point charge Q is located a distance d from the center of a grounded conducting sphere, as shown below. The conducting sphere has a radius a.

   (a) (5%) Determine the image charge (location and amount) that can be used to solve the electric field and electric potential outside the conducting sphere.

   (b) (5%) Explain why the total charge induced on the conducting ball must equal to the amount of the image charge.

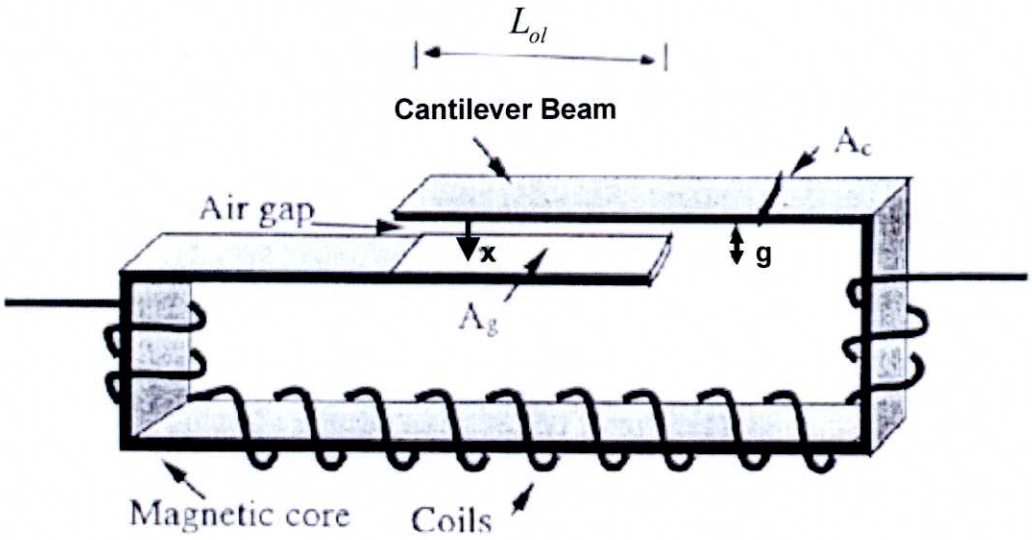


5. (20%) Consider a magnetic actuator shown in the following figure. The core is made of a magnetic material with a permeability μ and has constant cross area A_c. The gap, of area A_g and initial separation g, is shown on the structure. The total length of the core and the overlap length are denoted by L_c and L_{ol}, respectively. Please answer the following question:

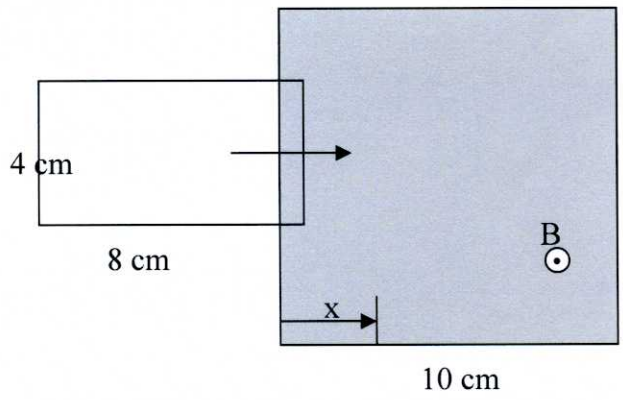
   (a) (5%) Neglect fringing fields, please represent the magnetic reluctance of the gap (R_{gap}) and of the magnetic core (R_{core}).

   (b) (5%) What is the total energy stored in the system including the core and the gap?

   (c) (10%) What is the magnitude of the force acting on the gap with respect to the gap variation x?



6. (20%) A uniform magnetic field B=200 mT extends over a square area (10 cm x 10 cm) with no field outside. A rectangular wire loop (8 cm x 4 cm) is moved through the field with constant velocity v. If the induced voltage is 2 mV, find out the loop velocity and the position x.



7. (20%) For a propagating wave, each component k_j $(j = x, y, z)$ takes the values satisfying $-|k|^2 \leqq k_j \leqq |k|^2$, where $k^2 = k_x^2+k_y^2+k_z^2 = (n\omega/c)^2$. When a TE wave is totally reflected from a dielectric interface, the field on side 1 forms a perfect standing wave, see Fig. 1. The maximum of the wave has phase φ(\phi) with respect to the interface; $\theta_g = 90^o-\varphi$, which is the Goos-Hanchen shift.

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(a) (5%) From the uncertainty relation of Fourier transform, show that the minimum variation in special coordinate Δr equals to $\lambda/4n$, which defines the resolution limit.

(b) (5%) Show that if one wants to have ***sub-wavelength resolution***, one possibility is to use total internal reflection.

(c) (5%) Solve the boundary value problem in the case when $\theta_1 > \theta_c$ and show that $|E_+^{(1)}| = |E_-^{(2)}|$, where θ_c is the critical angle.

(d) (5%) Develop an expression for cot θ_g in terms of θ_1 and θ_c.

Hint: at $z < 0$, assume that the superposition of incident and reflected waves is,

$$E_y = [E_+^{(1)} e^{-jk_z^{(1)} z} + E_-^{(1)} e^{+jk_z^{(1)} z}] e^{-jk_x x}$$

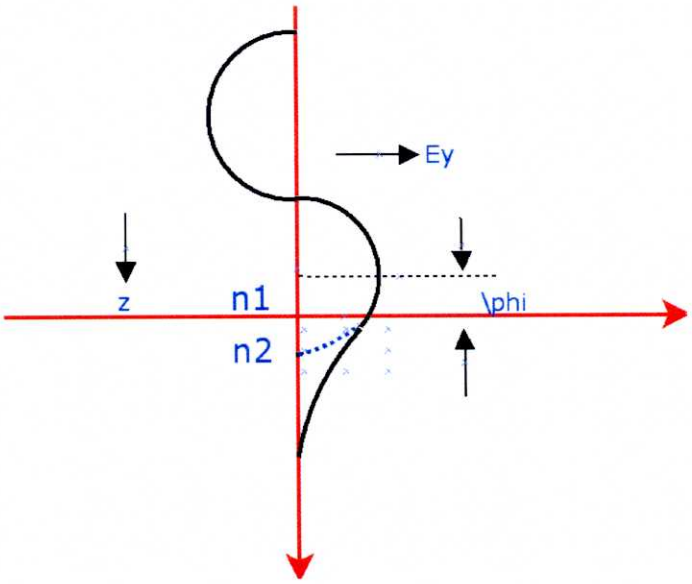

Fig. 1: Goos-Hanchen shift, with $n_1 > n_2$.