

(b) (4%) While placing an optical thin film (n=1.4) in one path of this Michelson interferometer, you observe seven periods shift in the interference pattern at the monitor. The optical source is 532nm. What is the thickness of the film?

4. (15%) A collimated 10-W 532 nm Gaussian beam with the beam waist (radius) of 1 cm is focused on the surface of a microscope cover glass. If the surface damage threshold is 10 MW/cm<sup>2</sup>, what's the shortest focal length lens can be used without causing damage. Based on the lens selection, what's the minimum value of the radius curvature of the focused Gaussian beam and where is it located? Any other thought?

5. (5%) Explain why light propagation can be considered as a low-pass (only low spatial frequency content can be transmitted) process based on the propagation transfer function.

6. (13%) In a simple (linear, homogeneous, isotropic) nondispersive medium, the polarization density  $\vec{P}$  (i.e. volume density of the induced electric dipole moment) is related to the external e-field  $\vec{E}$  via:

$$\vec{P} = \varepsilon_0 \chi \vec{E} \tag{1}$$

where  $\varepsilon_0$  is the permittivity of vacuum, and  $\chi (= \chi' + j\chi'')$  is the "constant" susceptibility of the medium. This means the bound electrons of the material can react to the external e-field instantaneously.

- (a) How to modify eq. (1) and  $\chi$  in the "frequency" domain (i.e. phasor relation) if the medium is simple but dispersive? (3 pts)
- (b) How to modify eq. (1) and  $\chi$  in the "time" domain if the medium is simple but dispersive? The result should show that the response of  $\vec{P}$  to  $\vec{E}$  is not instantaneous. (4 pts)
- (c) What are the physical meanings of  $\chi'$  and  $\chi''$ , respectively? (3 pts)
- (d) When the frequency of interest is close to resonance, i.e.  $v \approx v_0$ , the imaginary part of susceptibility can be approximated by a "Lorentzian" function:

$$\chi''(\upsilon) \propto \frac{1}{(\upsilon - \upsilon_0)^2 + (\Delta \upsilon/2)^2}$$

Please indicate the physical assumption that leads to this result (no formula is required). (3 pts)

7. (12%) Consider the vector phasor of the e-field of a time-harmonic uniform plane wave of angular frequency  $\omega$  propagating in the +z direction:

$$\vec{E}(z) = E_0 \left( 2\vec{a}_x - j\vec{a}_y \right) e^{-jkz}$$
, where  $E_0 = |E_0| e^{j\phi}$ 

- (a) If the plane of observation is located at z = 0, what are the x- and y-components of the timedependent e-fields  $E_x(t)$  and  $E_y(t)$ , respectively? (4 pts)
- (b) Describe the corresponding polarization state, including the trajectory and sense of rotation of the e-field. (4 pts)
- (c) Describe and justify a method that can change the polarization state of the lightwave. (4 pts)
- 8. (9%) The *mutual intensity* of an optical wave at points on the x axis is given by

$$G(x_1, x_2) = I_0 \exp\left[-\frac{(x_1^2 + x_2^2)}{W_0^2}\right] \exp\left[-\frac{(x_1 - x_2)^2}{\rho_c^2}\right]$$

where  $I_0$ ,  $W_0$ , and  $\rho_c$  are constants.

(a) Derive an expression for the normalized mutual intensity  $g(x_1, x_2)$  (3%) and sketch it as a function

of  $x_1 - x_2$  (3%).

(b) What is the physical meaning of the parameters  $I_0$ ,  $W_0$ , and  $\rho_C$ ? (3%)

9. (6%) Consider the following three basic hypotheses:

(a) For sufficiently small  $\Delta t$ , the probability of a single impulse occurring in the time interval t to

 $t + \Delta t$  is equal to the product of  $\Delta t$  and a real nonnegative function  $\lambda(t)$ ; thus  $P(1;t,t+\Delta t) = \lambda(t)\Delta t$ .

(b) For sufficiently small  $\Delta t$ , the probability that more than one impulse occurs in  $\Delta t$  is negligibly small

(i.e. there are no "multiple" events); hence  $P(0;t,t+\Delta t) = 1 - \lambda(t)\Delta t$ .

(c) The numbers of impulses in nonoverlappling time intervals are statistically independent.

Then by using the three fundamental hypotheses above show that the photocount statistics for light from a single-mode, amplitude-stabilized laser radiations obeys the *Poisson process*.

 (15%) Consider electromagnetic waves resonating inside a Fabry-Perot cavity as shown in the figure below. Apparently, there are multiple resonant frequencies (or wavelengths) v1, v2, v3, ....

The free spectrum range is  $\Delta v = \left(\frac{2L}{c}n_g\right)^{-1}$ , where ng is the group velocity index and c is the speed of light. Please answer the following questions:

$$\Delta v = \left(\frac{2L}{c}n_g\right)^2$$

(1) (10%) Please derive the equation
(2) (5%) Please give an intuitive explaining why the group velocity index is used instead of the phase velocity index.

