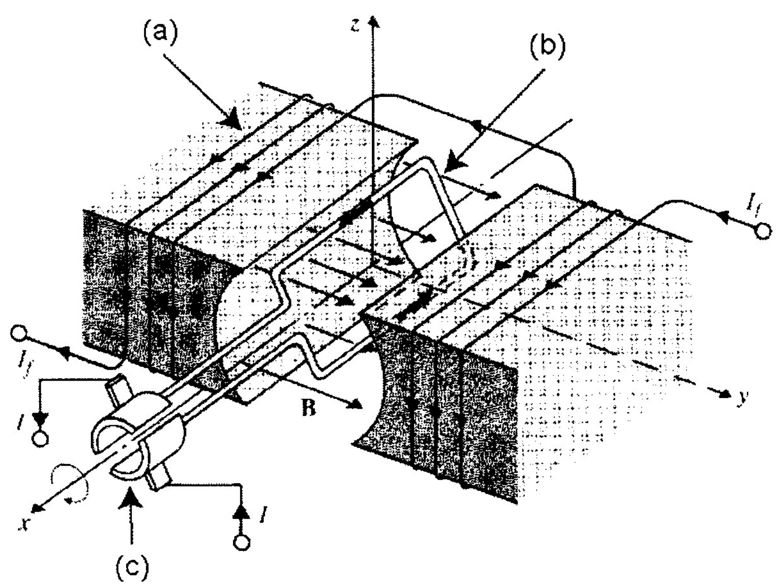
# 國立清華大學命題紙

九十四 學年度第二學期 光電工程研究所 博士班研究生資格考試

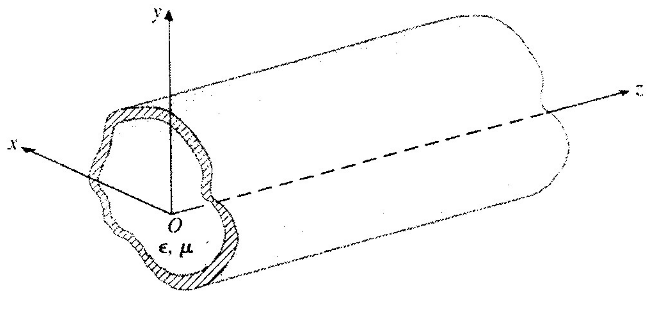
科目 電磁理論 科號 共 5 頁第 1 頁 *請在試卷(答案卷)內作答

1. Consider two coupled circuits, having self-inductances $L_1$ and $L_2$, that carry currents $I_1$ and $I_2$, respectively. The mutual inductance between the circuit is $M$.
   (a). Find the ratio $I_1/I_2$ that makes the stored magnetic energy $W_2$ a minimum. (5 %)
   (b) Show that $M \leq \sqrt{L_1 L_2}$. (5 %)

2. (14 %) Below shows the perspective view of a dc motor. Please clearly describe the principle of how this motor works by stating the functions and mechanisms of each of the parts (a), (b), and (c) as shown.




3. Consider *harmonic* wave (of single angular frequency $\omega$) propagation in a straight waveguide lying along the $z$-axis with arbitrary cross-section and filled with air (see figure below, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$). To analyze the guiding properties, we normally substitute $\vec{E}(x,y,z) = \text{Re}\{\vec{E}^0(x,y)e^{-j\beta z}\}$, $\vec{H}(x,y,z) = \text{Re}\{\vec{H}^0(x,y)e^{-j\beta z}\}$ into the vector Helmholtz's equations: $\nabla^2\vec{E} + k^2\vec{E} = 0$, $\nabla^2\vec{H} + k^2\vec{H} = 0$ ($k=\omega/c$, $c$ is light speed in vacuum), and try to solve the *guided modes* described by real $\beta$ values and corresponding field distributions $\vec{E}^0(x,y)$, $\vec{H}^0(x,y)$ in the presence of boundary conditions.

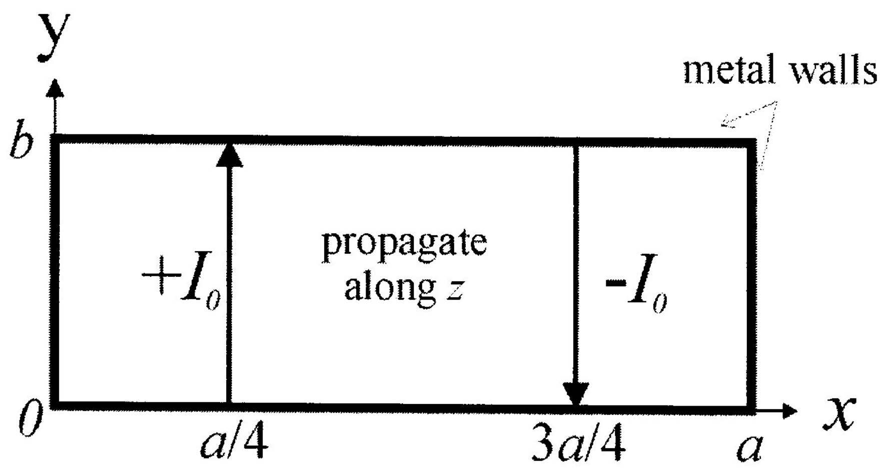



(a) (4%) What are the equations that govern $\bar{E}^0(x,y)$ and $\bar{H}^0(x,y)$, respectively?

(b) (4%) In Cartesian coordinates, results of (a) are actually six *scalar* equations governing $E_i^0(x,y)$, $H_i^0(x,y)$ ($i$=$x$, $y$, $z$), respectively. In practice, we can just solve $E_z^0(x,y)$, $H_z^0(x,y)$, and derive the remaining four by linear combinations of partial derivatives of $E_z^0$, $H_z^0$ (for example, $E_x^0 = \frac{-j}{k^2-\beta^2}\left(\beta\frac{\partial E_z^0}{\partial x} + \omega\mu\frac{\partial H_z^0}{\partial y}\right)$). Can you justify the above strategy? (Exact formulae of $E_x^0$, $E_y^0$, $H_x^0$, $H_y^0$ are NOT required)

(c) (6%) For TEM waves ($E_z^0$=0 and $H_z^0$=0), the strategy of (b) seems to result in trivial solutions $\bar{E}^0$=0 and $\bar{H}^0$=0, which prohibits TEM propagation along a waveguide. This conclusion, however, is NOT true for *two-conductor* waveguides, such as slab waveguides and coaxial cables. In these cases, how to evaluate the $\beta$ value(s) and the nonzero field components $E_x^0$, $E_y^0$, $H_x^0$, $H_y^0$ of **TEM** waves? Also, is there a *cut-off* frequency below which the TEM waves cease to propagate?

(d) (6%) Consider a rectangular metal-wall waveguide of dimension $a$×$b$. There are two thin wires located at $x$=$a$/4, and $x$=3$a$/4, driven by current signals $\pm I_0\cos(2\pi ft)$ (see figure below).




By (a-b), the field components of the corresponding $\mathbf{TE}_{mn}$ mode ($m,n \in$ integers) are:

$H_z^0 \sim \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$, $E_x^0 \sim \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$, $E_y^0 \sim \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$,

$H_x^0 \sim \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$, $H_y^0 \sim \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$, and cut-off frequency

$(f_c)_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$. Which TE mode is excited most efficiently if the current source frequency $f \gg c/a$ and $c/b$? Justify your answer. (Partial grades will be given if your description makes sense.)

4. Consider a plane wave propagating in the z direction with the electric field E = $a_y E_y(z,t)$, where $a_y$ is the unit vector in the y direction.

(a) (2%)Write the mathematical expression for $E_y(z,t)$.

(b) (9%) Prove the identity $\partial E_y / \partial z = -\partial B_x / \partial t$, where $B_x$ is the x component of the corresponding magnetic field of the wave.

5.(5%)A laser provides pulses of EM-radiation in vacuum lasting for $10^{-12}$ seconds. If the radiant flux density is $10^{20}$ W/m$^2$, determine the amplitude of the electric field of the beam.

6. An EM wave would induce an electric current on the surface of a good conductor.

(a) (5%)Explain why this would happen.

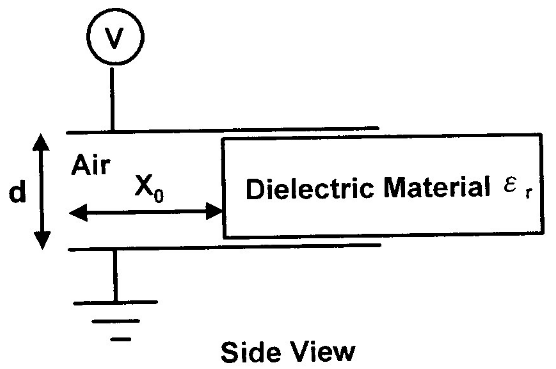
(b) (6%)Suppose a TE polarized plane wave is incident on a good conductor. Draw a figure to show how the electric current flows.

(c) (5%) Is there any force (no matter how small it is) exerted on the good conductor when this wave is incident? Explain why.

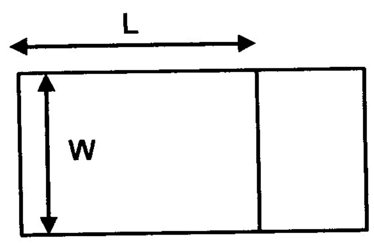
7. Consider a parallel plate inserted by a dielectric material ($\varepsilon_r > 1$) in between. The dimensions of the parallel plate are shown below. If the gap spacing between the parallel plate and the dielectric is neglected, please answer the following questions.

(i). If a voltage is applied on the parallel plate and the dielectric is movable, will the dielectric go left or right? No credit if there is no explanation. (12%)

(ii). Please derive the force as a function of the parameters given. Don't forget to mention the direction of force. (Suppose the fringe field is neglected.) (12%)




Side View




Top View