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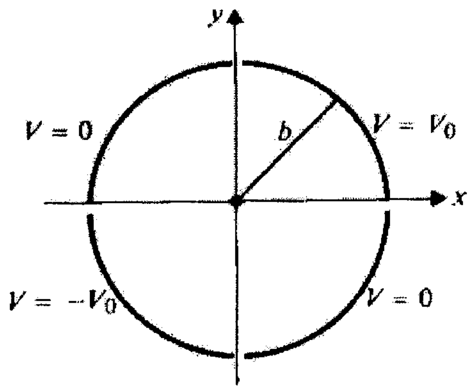
# 國 立 清 華 大 學 命 題 紙

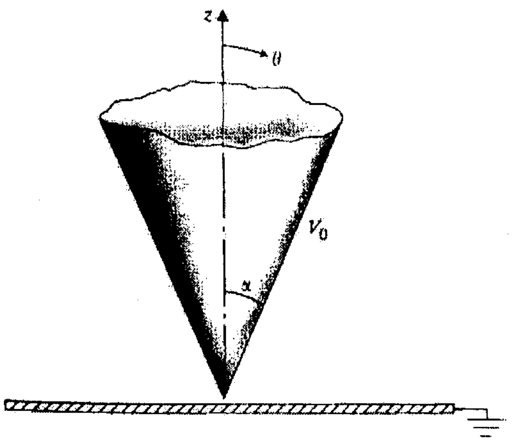
九十五 年度第一學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號____ 共 3 頁第 1 頁 *請在試卷(答案卷)內作答

1. (10 %) An infinitely long, thin conducting circular cylinder of radius b is split in four quarter-cylinders. The quarter-cylinders in the second and fourth quadrants are grounded, and those in the first and third quadrants are kept at potentials $V_0$ and $-V_0$, respectively. Determine the potential distribution both inside and outside the cylinder.




2. (15%) An infinite conducting cone of half-angle $\alpha$ is maintained at potential $V_0$ and insulated from a grounded conducting plane. Determine the potential distribution $V(\theta)$ in the region $\alpha < \theta < \pi/2$. Does the potential distribution V depend on the distance from the tip of the infinite cone? Why?




3 (5%) What are the advantages and disadvantages of using Maxwell's equations in *differential form* and *integral form*?

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4 (10%) For a plane wave propagating in a general direction,

$$\vec{E} = \vec{E}_+ e^{-i(\vec{k}\cdot\vec{r}-\omega t)},$$

$$\vec{H} = \vec{H}_+ e^{-i(\vec{k}\cdot\vec{r}-\omega t)},$$

where $\vec{k}$ is the propagation vector normal to the phase fronts.

(a) In isotropic media, show that the complex magnetic field vector $\vec{H}$ is perpendicular to $\vec{E}$.

(b) In isotropic media, show that the time-averaged power flow density vector points in the direction of $\vec{k}$.

(c) In anisotropic media, what vector is the complex magnetic field vector $\vec{H}$ perpendicular to?

(d) In anisotropic media, what direction does the time-averaged power flow density vector point to?

5. (10%) Sometimes it is more convenient to solve Maxwell's equations by introducing a vector potential, $\vec{A}$, and a scale potential $\Phi$,

$$\vec{B} = \nabla \times \vec{A},$$

$$\vec{E} = -\nabla\Phi - \frac{\partial A}{\partial t}.$$

(a) Show what is the *Lorentz gauge*.

(b) In a *non-uniform* dielectric, write down the wave equations for the vector potential, $\vec{A}$, and the scale potential $\Phi$.

6. (10%) A symmetric slab waveguide is found to support exactly five TE and five TM modes at an optical wavelength λ = 500 nm. Assume that dispersion of the waveguide material is negligible.

(a) How many TE and TM modes does it support at λ = 1 μm? (5%)

(b) Which mode among those at 500 nm and 1 μm wavelengths has the largest propagation constant? (5%)

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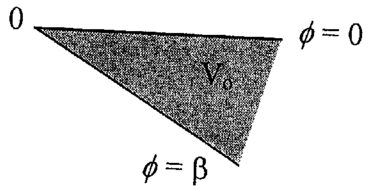
7. (15%) A symmetric slab waveguide has a core thickness of 2 μm. Ignoring the dispersion of the waveguide material, we find the indices to be $n_1 = 1.50$ and $n_2 = 1.46$.
   (a) Is this waveguide single moded or multimoded at $\lambda = 1.5$ μm and 1.3 μm? (5%)
   (b) What is the range of wavelength in which this waveguide is single moded? (5%)
   (c) If we want to make the waveguide to be single mode at both $\lambda = 1.5$ μm and 1.3 μm, how should we change the waveguide parameter? (A qualitative answer is sufficient.) (5%)

8. (16%) Consider a two-dimensional corner conductor as shown kept at potential $V_o$, the Laplace equation for the potential $\Phi(r, \phi)$ is $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \phi^2}=0$.
   (a) By separation of variables, find the general form of $\Phi(r, \phi)$. (3%)
   (b) Write the boundary conditions. (3%)
   (c) By the single-valued requirement on $\phi$, find the solution $\Phi(r, \phi)$. (3%)
   (d) By the requirement of $\Phi(0, \phi)$ = finite, find the form of $\Phi(r, \phi)$. (3%)




9. (9%) Extending the problem (8) to 3-dimensional corner conductor with its edge located at z-axis,
   (a) write the Laplace equation for $\Phi(r, \phi, z)$. (3%)
   (b) find the general form of $\Phi$ by using separation of variables. (3%)
   (c) find the boundary conditions. (3%)