國立清華大學命題紙

九十三 學年度第二學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號 ＿＿ 共 4 頁第 1 頁 *請在試卷(答案卷)內作答

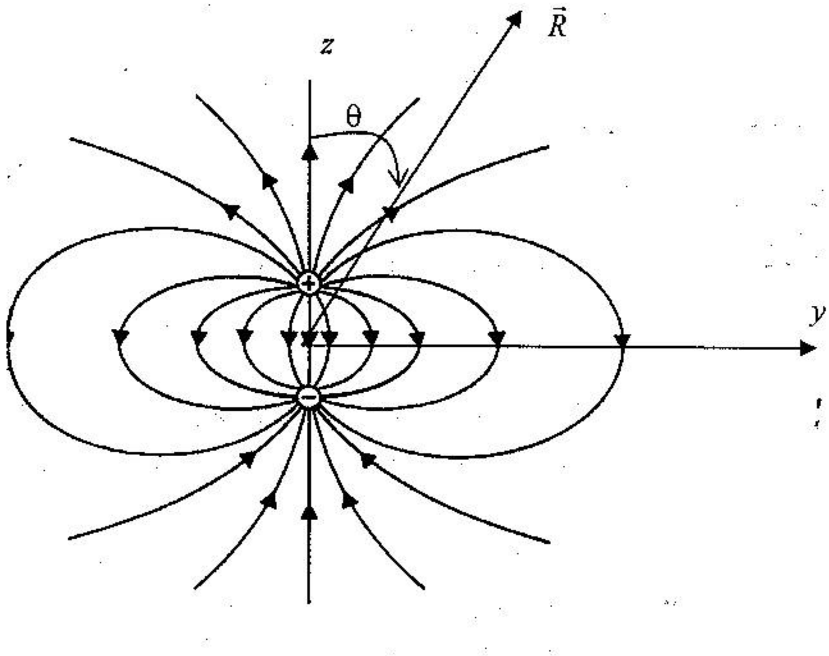
1. (15%) A spherical conductor of radius r_1 carries a charge Q. It's surrounded by a dielectric material of permittivity ε, out to radius r_2. Find the electric field in all regions (out to infinity). Find the electrostatic energy stored in this configuration. Calculate the total capacitance (considering the outer conductor surface is an imaginary spherical shell of infinite radius.

2. (10%) A very long cylindrical shell (inner radius r_1 and outer radius r_2) has the following charge distribution: $\rho = \frac{1}{r}$ (C/m^3), $r_1 < r < r_2$. Find the electric field everywhere.

3. (25%) The following plot shows the electric field lines of an electric dipole in the spherical coordinate system. The corresponding Cartesian coordinate system is also shown below. The far-field electric-field lines are drawn from the field solution

$$\vec{E} = \frac{p}{4\pi\varepsilon_0 R^3}(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta),$$

where p is the electric-dipole moment, ε_0 is the vacuum permittivity, and $\hat{a}_R, \hat{a}_\theta$ are the unit vectors along the R, θ directions of the spherical coordinate system, respectively.



a) Draw the equipotential lines overlapped to the electric lines in the yz plane. You MUST explain the theoretical reason or the physical basis of your drawing. No explanation, no credit. (5%)

b) A circular current loop of radius b carrying a current $I\hat{a}_\phi$ is placed in the xy plane with its center coinciding with the origin of the coordinate system. This current loop gives the far-field magnetic dipole field similar to the far-field electric dipole field, given by

$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0 m}{4\pi R^3}(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta)$$

where m is the magnetic dipole moment, $\vec{A} = \hat{a}_\phi \frac{\mu_0 m}{4\pi R^2}\sin\theta$, and μ_0 is the vacuum permeability.

i. Draw the far-field and near-field magnetic field lines of this magnetic dipole in the yz plane. In your plot, indicate the location and direction of the current flow and the field lines. (5%)

ii. Draw the field lines and directions of the magnetic vector potential $\vec{A}$ in a plane defined by z = constant. Let the density of lines in your plot be proportional to the field strength of $\vec{A}$. (5%)

iii. What is the physical meaning of the curl of a vector $\vec{C}$ or $\nabla \times \vec{C}$ in space? Do not write any mathematic formula but just provide a physical explanation to this mathematic operation. (5%)

iv. From the physical definition of the curl operator, explain why $\vec{A} = 0$ but $\vec{B}_R = \frac{\mu_0 m}{4\pi R^3}\hat{a}_R 2\cos\theta$ is a maximum at $\theta = 0$. (Isn't it unusual that the B field can be a maximum at a location where the magnetic vector potential is zero?) (5%)

Note: for the following four questions, detailed explanations are required. Sketches are encouraged. No credits will be given for answering only yes or no.

4. (Total 6%)

a) What happen when a uniform plane wave in medium 1 is incident on a less dense medium 2 if the incident angle is larger than the critical angle? (2 %)

b) Is there any power transmitted into medium 2? Where does the energy go? (2 %)

c) Quartz windows set at the ends of a laser tube are used to control the polarization of an emitted light beam. How should the windows be arranged? What kind of light in terms of polarization can be obtained? (2 %)

5. (Total 8%)

a) Explain the physical meanings of group velocity (2 %) and phase velocity (2 %).

b) Can the group velocity be larger than the phase velocity? If yes, in what situation? If not, why not? (2 %)

c) Can the group velocity be larger than the speed of light in vacuum? Can the group velocity be negative? Explain your answer in detail. (2 %)

6. (Total 8%)

a) Explain the principle of a speed gun used by police that detect the speed of moving vehicles. What can be done to make it detects more accurately? (4 %)

b) Instead of using a narrowband signal which has a particular frequency, can the detection be done if the signal wave is a broadband signal? (eg. 1 GHz bandwidth centered at 10 GHz) What happen to the spectrum of the returned signal? (2 %)

c) How about if a baseband signal is used? (eg. 1 GHz bandwidth span from DC to 1 GHz) What happen to the spectrum of the returned signal? (2 %)

7. (3 %) The electromagnetic waves radiated by AM broadcast stations from their antenna towers are linearly polarized with the E-field perpendicular to the ground. Television signals are linearly polarized in the horizontal direction. The waves radiated by FM broadcast stations are generally circularly polarized. How should the receiving antennas be arranged for maximum reception for each of these three different cases?

8. (Total 25 %) For a dielectric slab waveguide shown below, assume no dependence on x-direction and an optical wave propagates in $+z$-direction:

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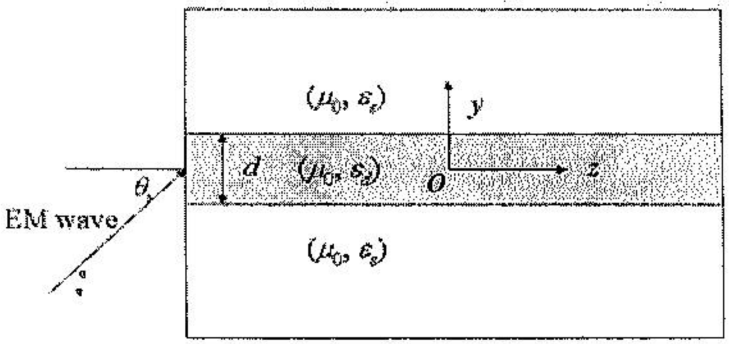
科目 電磁理論 科號 共 4 頁第 4 頁 * 請在試卷(答案卷)內作答



a) Can such a slab waveguide support a TEM wave? Why? (6 %)

b) For such a slab waveguide, if we wish the optical waves to be "guided" in the slab, we can expect the total internal reflections (TIRs) occur at the boundaries of $y = \pm\, d/2$. To achieve TIR, what's the required relation between ε_d and ε_s? (2 %)

c) Following b), what is the maximal incident angle, θ_i, such that TIRs are guaranteed at all interfaces? (5%)

d) In b) and c), we viewed the optical wave in the waveguide as an "optical ray." Now if we view the optical wave as a "wave," please find the required relation of the phase constant β, ε_d and ε_s, so that the wave can be "guided" in the slab waveguide. (6 %)

e) Following d), when we know the frequency of this optical wave, we may apply the following figure to obtain the possible solutions for odd TM mode graphically. If we wish to design a slab waveguide such that "single mode" operation is achieved at a fixed frequency, describe and explain your approaches to achieve this goal. (6 %)