## 博士班研究生資格考試 一百學年度第二學期 \*請在試卷(答案卷)內作答

Consider the ABCD matrix for an optical system:

(a) (6%) If matrix element A=0, what is the special characteristic of this system? Draw a simple picture showing the relationship between input and output rays along with your description.

(b) (4%) Now what happens when B=0?

A white light source is filtered with a monochromator and sent into a Michelson interferometer. The transmission of the monochromator  $T(\lambda)$  is

$$T(\lambda) = \begin{cases} 1, & (\lambda_0 - \frac{1}{2}\delta\lambda) < \lambda < (\lambda_0 + \frac{1}{2}\delta\lambda) \\ 0, & otherwise \end{cases}$$

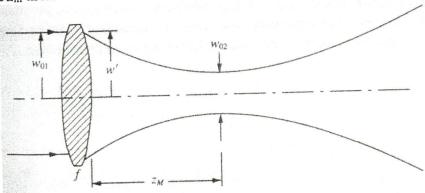
with a width of  $\delta \lambda = 0.1$  nm centered at  $\lambda_0 = 795$ nm.

(a) (4%) Please find the transmission function  $T(\omega)$  of the monochromator in terms of the optical frequency  $\omega$ , and find its width  $\delta\omega$ .

(b) (5%) Please find the interference intensity I(x) at the output of the interferometer as a function

of the path length difference x (consider monochromatic wave).

- (c) (6%) A white light source, which is filtered with a monochromator with transmission function  $T(\lambda)$  defined above, is sent to the interferometer. Please find the interference intensity I(x) at the output of the interferometer as a function of the path length difference x.
- 3. (10 %) The figure below shows the focusing of a Gaussian beam by a lens with a focal length  $f_{
  m c}$ where  $w_{01}$  is the spot size right before the lens and  $w_{02}$  is the minimum focal spot size achieved at  $z_m$  after the lens. Please calculate the following in terms of f,  $w_{01}$ , and  $l_0$ .



- (a) Where does the minimum focal spot size occur?  $(z_m = ?)$
- (b) What is the minimum spot size at the focus?  $(w_{02} = ?)$
- 4. (a) (7%) A transparency slide, as shown in the left figure, is placed at the object plane of a 4-f imaging system, and a spatial filter, as shown in the right figure, is placed at the Fourier plane. Sketch the diffraction pattern at the image plane qualitatively. Explain your answer in detail.





(b) (8%) Explain the basic design principles for the safety grid on a micro-wave oven door.

5. (a) (2%)Derive the dispersion relation, i.e., how  $\omega$  (temporal frequency) is related to eta (spatial

frequency), from the Maxwell equation in free space as shown below.

$$\nabla^2 \mathbf{\Psi} - \frac{1}{c^2} \frac{\partial^2 \mathbf{\Psi}}{\partial t^2} = 0$$

(b) (2%)Do the same for a Schrodinger-like equation as shown below.

$$\nabla^2 \mathbf{\Psi} - \frac{1}{ic} \frac{\partial \mathbf{\Psi}}{\partial t} = 0$$

- (c) (3%)Explain, when given a spatially localized wavepacket, which of the above two equations is diffusive or diffusionless and why. Note that diffusion refers to the fact that a spatially localized wavepacket will expand as a function of time.
- (d) (5%) Consider a metallic slab waveguide with thickness d as shown below, and derive its dispersion relation. (3%) Under what condition the derived dispersion relation becomes diffusive or diffusionless?
- (10%) A Wollaston prism (WP) is made of two cemented uniaxial crystals A, B with the same 6. ordinary and extraordinary indices  $n_{\scriptscriptstyle 0}$ ,  $n_{\scriptscriptstyle e}$  but orthogonal optic axes (Figure 1). Explain what will happen at the output when an elliptically polarized light is normally incident on such a WP. Justify your answer.

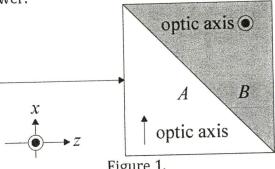
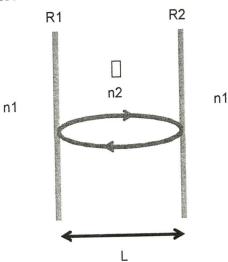


Figure 1.

- 7. Consider electromagnetic waves resonating inside a Fabry-Perot cavity as shown in the figure below. R1 and R2 are the reflectivity at the two end of the cavity. a is the attenuation coefficient and n1, n2 are the refractive indices.
- (1) (5%) What is the finess?
- (2) (5%) What is the photon life time?
- (3) (5%) What is the quality factor?



8. (a) (5%) Show that the **power spectral density**, S(v), defined as  $S(v) = \lim_{T \to \infty} \frac{1}{T} < |V_T(v)|^2 >$ , is the Fourier transform of the autocorrelation function,  $\,G( au)$  , i.e.

$$S(v) = \int_{-\infty}^{\infty} G(\tau) \exp(-j2\pi v \tau) \ d\tau,$$

where 
$$V_T(v) = \int_{-T/2}^{T/2} U(t) \exp(-j2\pi vt) dt$$
, and  $G(\tau) = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} U^*(t) U(t+\tau) dt$ .

(b) (5%) The *mutual intensity* of an optical wave at points on the x axis is given by

$$G(x_1, x_2) = I_0 \exp\left[-\frac{(x_1^2 + x_2^2)}{W_0^2}\right] \exp\left[-\frac{(x_1 - x_2)^2}{\rho_c^2}\right],$$

where  $I_0$ ,  $W_0$ , and  $\rho_C$  are constants. Derive an expression for the normalized mutual intensity  $g(x_1,x_2)$  and sketch it as a function of  $x_1-x_2$ . What is the physical meaning of the parameters  $I_0$ ,  $W_0$ , and  $\rho_C$ ?