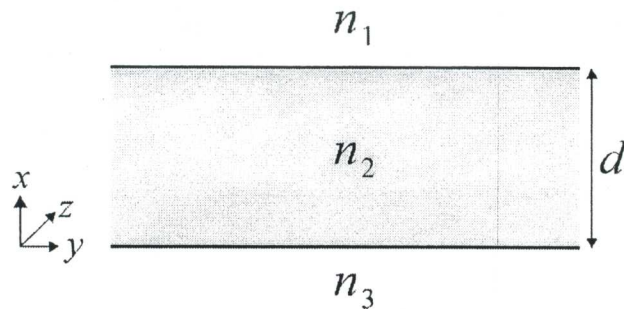


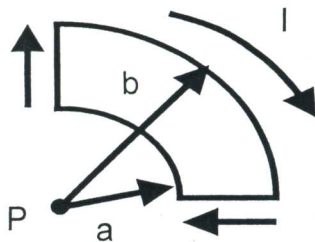
(I) Consider a dielectric slab waveguide of the following form:



where refractive index  $n_2 > n_1, n_3$ , and the guided wave propagates in the  $z$ -direction.

- (a) (10%) The guiding mechanism can be explained by total internal reflection in ray optics, where an incident angle  $\theta$  greater than the critical angle  $\theta_c$  is required. Does that mean there will be no “cut-off” as long as optical beam is normally incident ( $\theta=90^\circ$ )? Why?
- (b) (10%) To make a bio-sensor, we dissolve some optically sensitive bio-targets in some solvent of index  $n_s$ , and deposit them on the  $n_1$ - $n_2$  interface. Which type of solvent ( $n_s > n_2$ , or  $n_s < n_2$ ) should be used to get better sensitivity? Why?

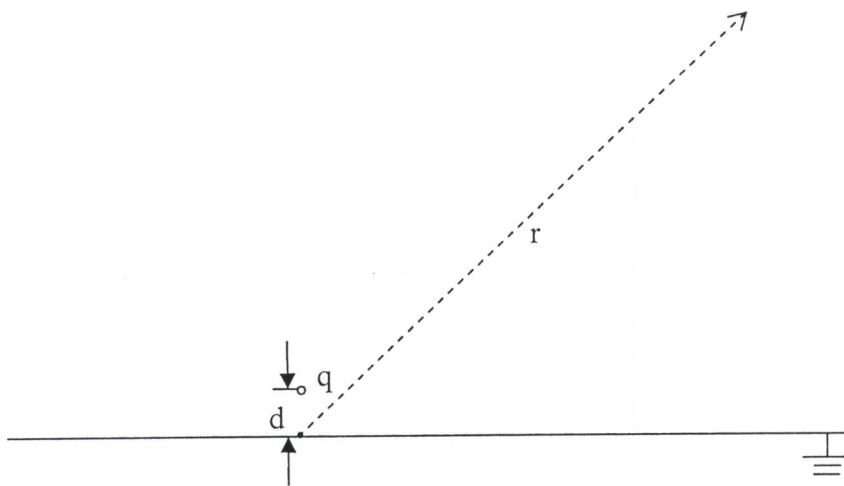
(II) (15 %) Find the magnetic field at point P for the following steady current configuration



(III) For the following 5 questions, scores are given based on your ability in reasoning but not solely based on your ability in providing correct answers. To get a good score, you are encouraged to express your view as much and as reasonable as possible.

1. (5%) We learn about plane waves in the electromagnetics course. What is the “plane” referred to?
2. (5%) Given an amplitude of some finite value, a plane wave in free space carries an infinite amount of energy and is nonphysical. If so, why do we study plane waves so extensively in the electromagnetics course?
3. (5%) You were told in most situations that the electric field of an electromagnetic wave is perpendicular to the propagation direction of the wave. Give at least one example in which some electric field component is along the energy propagation direction of an electromagnetic wave. Offer your explanation for this kind of situation.
4. (5%) Is it possible that the rotation frequency of the electric-field vector of a circularly polarized electromagnetic wave in space is different from  $2\pi c / \lambda$ , where  $c$  is the speed of the wave and  $\lambda$  is the wavelength of the wave.
5. (5%) Why does a metal reflect most of the sun lights? Consider a metal as plasma. Under what situation could an electromagnetic wave propagate in a metal?

(IV) (12%) As shown in the following figure, a positive charge  $q$  is located  $d$  above a large conducting plate that is at potential zero. Show that the electric potential  $V$  in the far field ( $r \gg d$ ) is proportional to  $r^{-2}$ .



(V) (15%) Determine the electric field everywhere induced by a charged dielectric sphere of radius  $R_0$ . The charge density  $\rho(r)$  is distributed as a constant  $\rho_0$  from origin to  $R_0/2$  and  $(\rho_0 R_0)/2r$  from  $R_0/2$  to  $R_0$ . If this sphere is concentrically enclosed in a conducting sphere of

radius  $2R_0$ , what will be the electric field outside?

(VI) (13%) A plane wave polarized in the Y axis is incident upon Y-Z plane at an incidence angle  $\theta$ , as shown below. Use Maxwell's equation to find the magnetic field of the wave.

