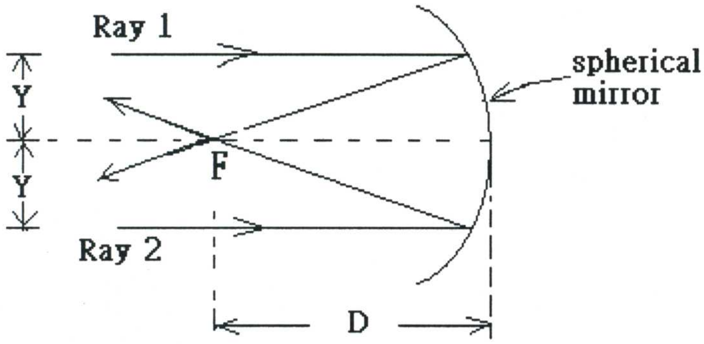
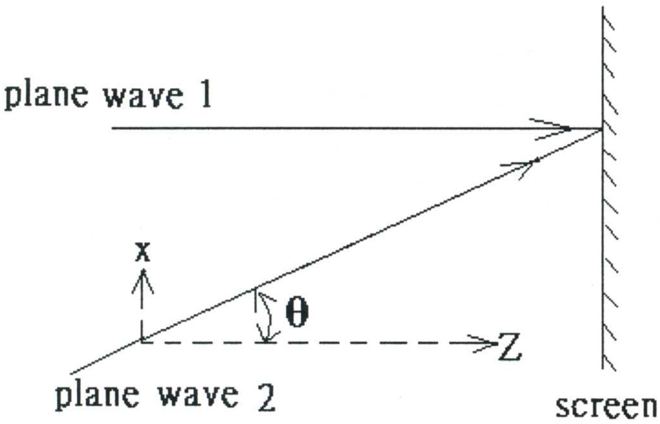
1. The ray-transfer matrix (i.e., the ABCD matrix) for a free space propagation is $\begin{bmatrix}1 & d\\0 & 1\end{bmatrix}$, and that for the reflection from a concave spherical mirror is $\begin{bmatrix}1 & 0\\-2/R & 1\end{bmatrix}$, where R is the radius (R>0).

   Two parallel rays separated by 2Y are incident upon a concave spherical mirror, as shown below. Suppose they will converge at point F. Determine the required D. (10%)



2. Two monochromatic plane waves with the same optical frequency are incident upon a screen, as shown below.
   (A) Write the wave expression for each wave in the form of $e^{j(fx+gz)}$, where f and g may or may not depend on θ. (5%)
   (B) The two waves interfere with each other and form a fringe pattern on the screen. Write the mathematic expression for the fringe pattern. (5%)



3. A 1-W frequency-doubled Nd:YAG laser (532 nm) is used to produce local

heating for laser machining. The beam waist radius w_0 exiting laser is 6 mm. If the minimum required irradiance (intensity) is 25 W/cm^2, what's the largest necessary spot size? What's the longest focal length of a focusing lens we have to provide? If the peak irradiance we can obtain is 50 W/cm^2, what's the focusing tolerance in the propagation direction (assuming the focused spot size is not restrained)?(15%)

4. (10 %) The susceptibility of a resonant medium can be expressed as:

$$\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + j\nu\Delta\nu}$$

where ν_0 is the resonance frequency, χ_0 is the low-frequency susceptibility.

(a) Please draw the real and the imaginary part of the susceptibility $\chi'(\nu)$ and $\chi''(\nu)$. What happens to the susceptibility of a medium when it is pumped to population inversion?

(b) Show that the medium acts like free space at frequencies well above resonance.

(c) For a dispersive material, can it be transparent to light (no absorption)? Why?

(d) Please show that, when near resonance, the ratio between the real part and the imaginary part of the susceptibility is $\frac{\chi'(\nu)}{\chi''(\nu)} = 2\frac{\nu - \nu_0}{\Delta\nu}$.

5. (10 %)

(a) Show that the coherence time τ_c is related to the spectral width $\Delta\upsilon_c$ by the simple inverse relation $\tau_c = 1/\Delta\upsilon_c$ for a spectral density with a rectangular spectral profile.

(b) What are the coherence time τ_c and the spectral width $\Delta\upsilon_c$ of an ideal monochromatic light?

(c) For light that is already emitted out from a light source, can one increase the coherence time of the light? If yes, how? If no, why not?

6. Use Jone's calculus to analyze the polarization state of a light passing through an analyzer then a quarter wave plate. Draw the intensity variation when the quarter wave plate rotates. (8%)

7. Describe and explain in detail, with the tool of normal surface (k-surface) construction, the phenomena of double refraction. (12%)

8. A linear cavity consisting of two infinite plane mirrors with field reflectance r_1, r_2 separated by a distance d in vacuum (Fig. 1).

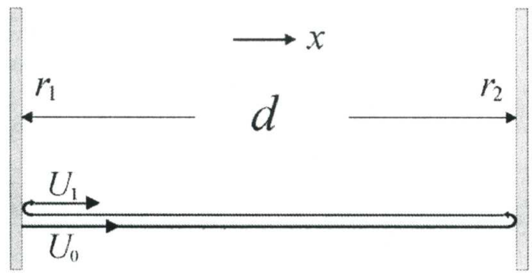


Fig. 1

(a) (3%) Determine the resonance frequencies ν_n and wavevectors $\vec{k}_n$.

(b) (3%) Determine the number of resonant wavevectors (modes) $N(\nu)$ between an infinitesimal frequency range $[\nu,\ \nu+d\nu]$, if $d >> \lambda = c/\nu$. Note $M(\nu) \equiv \dfrac{2N(\nu)}{d \cdot d\nu}$ is the mode density of the linear cavity (the factor "2" represents polarization degeneracy).

(c) (3%) Consider a time-harmonic wave $U_0 \exp(j2\pi\nu t)$ reflected back and forth between the two mirrors. What is the consecutive phasor U_1 after one round trip?

(d) (3%) What is the intensity in the resonator $I = |U|^2$, where $U = \sum_{i=0}^{\infty} U_i$?

(e) (5%) Roughly sketch the resonance spectrum $\left|\dfrac{U(\nu)}{U_0}\right|^2$, and explain how to use this cavity as a spectrometer.

(f) (3%) In thermal equilibrium at temperature T, a two-level system with energy difference $E_2 - E_1 = h\nu$ has an average number of photon $\bar{n} = \dfrac{1}{\exp(h\nu/k_B T) - 1}$, where k_B is the Boltzmann constant. What is the average energy $\bar{E}$ of a blackbody radiation mode of frequency ν at temperature T?

(g) (5%) Roughly sketch the blackbody radiation spectrum from a linear cavity at temperature T. Justify your answer.