

1. (10 %) Please derive the ray transfer matrix \mathbf{M} of a “telescope” (see Fig. 1).

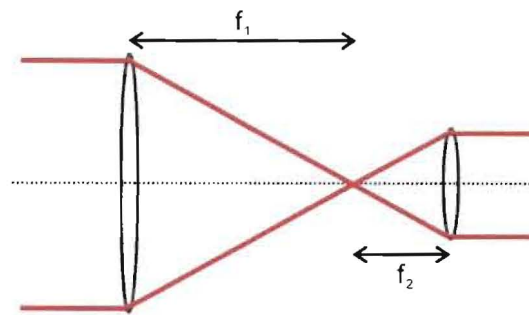
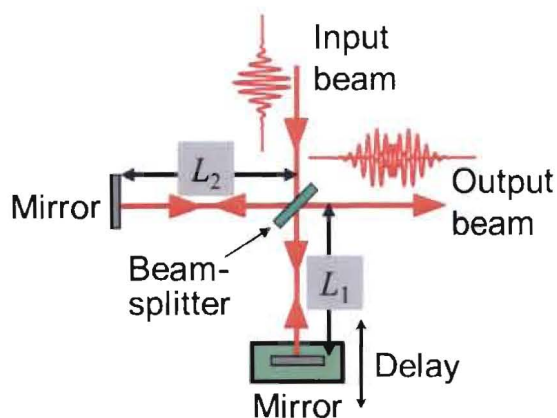


Fig. 1

2. (15%) Interferometers are often used in displacement detection.

- What type of interferometer is shown in the following figure on the left? (2%)
- If the input beam has a wavelength of $\lambda_0 = 1 \mu\text{m}$, how many fringes will you observe in the output if the distance between the beamsplitter and the mirror with variable delay (L_1) is increased by $10 \mu\text{m}$? (2%)
- What are the conditions to have the highest contrast of the fringes in the output? (2%)
- What do you expect to see in the output if the input beam is a white light? (2%)
- What interference features do you expect to observe on a film made of liquid dishwashing soap? What would be the differences when it is placed vertically and horizontally? Please draw and explain. (7%)



3. (10%) A 1000 W continuous laser produces a Gaussian beam of wavelength $\lambda = 1064 \text{ nm}$ with waist radius $W_0 = 0.5 \text{ mm}$. Design a single-lens system for

focusing the light to a peak intensity of 5 GW/cm^2 . What is the shortest focal-length lens that may be used?

4. (15%) (a) (7%) Please describe and explain the Abbe imaging theory. (b) (8%) Please design an experiment to perform the function of optical correlation.

5. (15%) Consider a gain medium with susceptibility expressed as follow:

$$\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + j\nu\Delta\nu}$$

(1) (7%) Please plot the real part of the susceptibility near the resonant frequency

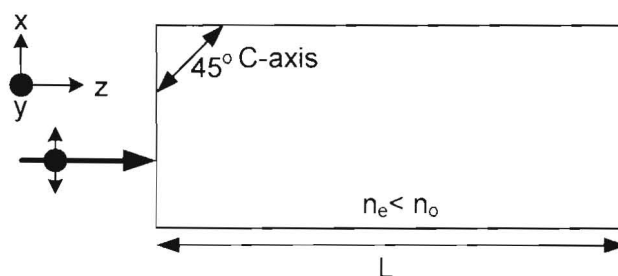
(2) (8%) Please plot the imaginary part of the susceptibility near the resonant frequency

6. (10%) Assume you are given a birefringent crystal as shown below.

a. (2%) What is the e-field direction for the ordinary-ray?

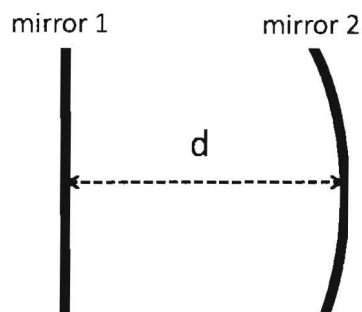
b. (2%) Plot the k-surfaces at the input air-crystal boundary.

c. (6%) Plot the k-direction, phase front, and the Poynting vector direction for the E-ray.



7. (15%) An optical cavity consisting of two mirrors is shown below. The radius of curvature of mirror1, mirror2, and the cavity length are R_1 , R_2 , d , respectively.

The refractive index inside the cavity is 1.



(a) (6%) Starting from the resonant condition, please derive an expression of the resonant frequencies of the Gaussian beam with $TEM_{\ell m}$ of the cavity.

Denote the longitudinal mode number as q .

(b) (5%) Given $d/R_2=0.5$, please derive an expression of the resonant frequency of $TEM_{\ell m q}$, where ℓ and m are the transverse mode numbers, and q is the longitudinal mode number.

(c) (4%) From (b), for the same longitudinal mode number, how many transverse modes can oscillate inside the cavity?

8. (10%) Plot the photon number distribution, $P(n) = |\langle n|\varphi\rangle|^2$, as a function of photon numbers for the state, $|\varphi\rangle$

(a) Number state, $|\varphi\rangle = |n = 3\rangle$;

(b) Coherent state, $|\varphi\rangle = |\alpha = 3\rangle$;