國立清華大學命題紙

九十 學年度第二學期 電機工程 學系 博士班研究生資格考試

科目 電磁理論 科號 ______ 共 3 頁第 1 頁 ＊請在試卷(答案卷)內作答

1. The rectangular cavity resonator is a box comprising the region 0 < x < a, 0 < y < b, and 0 < z < c, and bounded by perfectly conducting walls on all of its six sides. The time-varying electric and magnetic fields inside the resonator are given by:

$$\vec{E} = \hat{y}E_0 \sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos(\omega t) \qquad \text{(V/m)}$$

$$\vec{H} = \hat{x}H_{01}\sin\frac{\pi x}{a}\cos\frac{\pi z}{d}\sin(\omega t) - \hat{z}H_{02}\cos\frac{\pi x}{a}\sin\frac{\pi z}{d}\sin(\omega t) \qquad \text{(A/m)}$$

where E_0, H_{01}, and H_{02} are constants. Find the surface charge density ρ_s and the surface current density $\vec{J}_s$ on all six walls, assuming the medium inside the box to be a perfect dielectric of $\varepsilon = 4\varepsilon_0$. (15%)

2. An air-dielectric line of characteristic impedance $R_0 = 100$ (Ω) and length ℓ is terminated by a load impedance of $Z_L = 60 + j80$ (Ω).

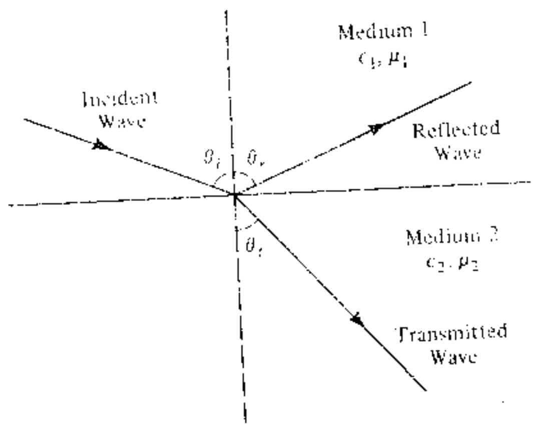
   (a) Find the reflection coefficient Γ at the load. (5%)
   (b) Find the standing wave ratio on this transmission line. (5%)
   (c) For a frequency $f = 3$ (GHz) and the length $\ell = 2.5$ (cm), find the input impedance of the line and the reflection coefficient Γ at the input end of the line. (5%)
   (d) For a frequency $f = 1$ (GHz) and the length $\ell = 15$ (cm), find the input impedance of the line and the reflection coefficient Γ at the input end of the line. (5%)

3. Consider a uniform plane wave incident obliquely on a plane boundary between two different perfect dielectric media at an angle of incidence θ_i to the normal to the boundary.

(i) Please calculate the reflection and transmission coefficient for TE polarization. (8%)

(ii) Which states of polarization (TE or TM) can have zero refection? Please clearly answer this question but WITHOUT using any mathematical derivations. (4%)

(iii) Consider medium 1 as a high-index material and medium 2 as a low-index material. We have total internal reflection. Can the incident field partially appear in medium 2 now? Is there any contradiction if you find some field in medium 2? (3%)



4. Consider three transmission lines, two-parallel lines, coaxial cables and metallic hollow waveguides. Please draw the schematic loss versus frequency (both with magnitudes) of these three waveguides. Please also explain why the characteristics of these curves given in your answer sheet? (10%)

5. In some real applications, we assemble several antennas as an array instead of using a single powerful antenna. Please explain the reason and working principle of arrayed antennas. (8%)

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6. Write down the Ampere's law modified by Maxwell and the continuity equation between current density **J** and charge density ρ_n. Then Show that the modified Ampere's law is consistent with the continuity equation. (10%)

7. Write down the electric field intensity **E** in terms of time-harmonic potentials **A** and Φ. Also write down the Lorenz gauge between the potentials. Then derive Gauss's law of electric field, one of Maxwell's equations. (10%)

8. The Biot-Savart law, which relates the magnetic field **B** directly with a time-harmonic current density **J**, reads

$$\mathbf{B}(\mathbf{r}) = \int \left[ak_0 - \frac{1}{R}\right] G(R)\hat{R} \times \mathbf{J}(\mathbf{r}')dv',$$

where Green's function $G(R) = e^{-jk_0R}/4\pi R$ and $R = |\mathbf{r} - \mathbf{r}'|$. This relation can be derived from the magnetic vector potential **A**, which in turn is related to the field **B** and to the density **J**. Derive the Biot-Savart law from the potential **A** and find constant a. (12%)

Formula for Reference

Helmholtz's equation: $\nabla^2\Phi(\mathbf{r}) + k_0^2\Phi(\mathbf{r}) = -\frac{1}{\epsilon_0}\rho_n(\mathbf{r})$