1. A fundamental Gaussian laser beam of power $P = 1$ W at a wavelength of $\lambda = 532$ nm is focused to a small spot radius of $w_0 = 10\mu m$ at its beam waist.

(1) Show that the intensity profile can be expressed as the following Gaussian function:

$$I(r,z) = I_0(z)\exp\left[-\frac{2r^2}{w^2(z)}\right],$$

where $I_0(z) = I_0\left[\frac{w_0}{w(z)}\right]^2$ and $w(z)$ are the respective peak intensity and the radius of the beam at the longitudinal location z, and $r = (x^2 + y^2)^{1/2}$ is the transverse radial distance. (2 %)

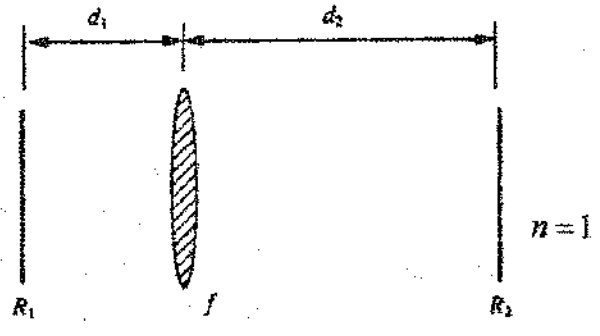
(2) Express the power, P, of this Gaussian beam as a function of its peak intensity $I_0(z)$ and its spot size $w(z)$ at any location z. Does the power P vary with z? (2 %)

(3) What is the peak intensity I_0 at the beam waist? (2 %)

(4) What is the divergence angle of the beam? (2 %)

(5) What is the spot size and peak intensity at a distance of 10 m from the beam waist? (2 %)

2. For the cavity shown below, the Hermite-Gaussian beam parameters are given by $z_{01} = \pi w_{01}^2 / \lambda_0 = 6$ cm and $z_{02} = 35$ cm with $d_1 = 24$ cm, $d_2 = 70$ cm, $R_1 = 0.98$, $R_2 = 0.93$, and a scattering loss of 5% / pass / surface. The wavelength region of interest is 514.5 nm.



(1) Find a formula for the resonant frequencies of the $TEM_{m,p,q}$ modes in terms of d_1, d_2, z_{01}, and z_{02}. (2 %)

(2) What is the frequency difference between the $TEM_{0,0,q}$ mode and $TEM_{1,0,q}$ mode? (2 %)

(3) What is the photon lifetime? (2 %)

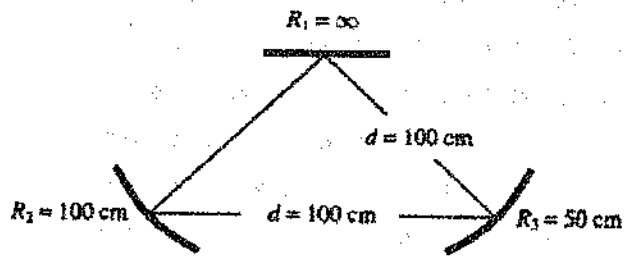
(4) What is the quality factor Q? (2 %)

(5) What is the Finesse F? (2 %)

3. Is the cavity shown below stable? Why?

(1) Find the ABCD matrix for the unit cell (3 %)

(2) Applying the stability criteria (2 %)



4. Show that a parabolic mirror will focus all incident rays parallel to its axis to a single point on axis. (15%)

5. Derive an expression for the intensity I of the superposition of two plane waves of wavelength λ traveling in opposite directions along the z axis. Sketch I vs. z. (10%)

6. In the case of gas system, the cross section at line center for a Lorentzian - lineshape transition between energy states E_1 and E_2 is given by

$$\sigma_o = \frac{\lambda^2}{2\pi} \frac{1}{2\pi t_{sp} \Delta\nu}$$

(1) Determine the wavelength λ. (3%)

(2) What is the physical meaning of t_{sp}? (4%)

(3) Explain the factors that affect the value of $\Delta\nu$. Which factor is the most important when the gas is under very low pressure? (4%)

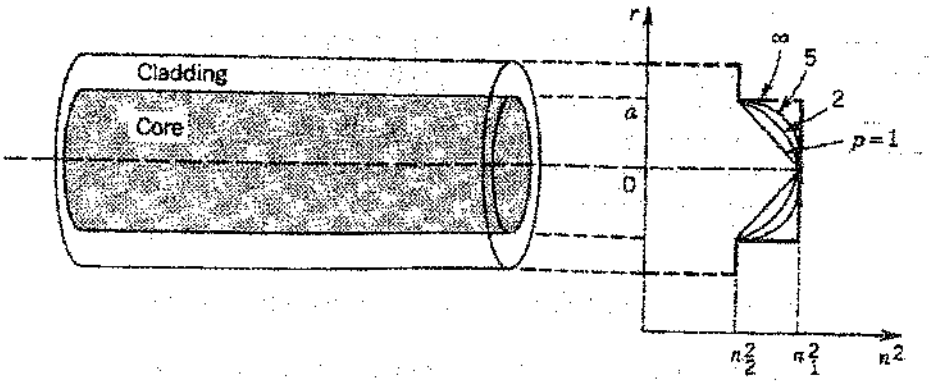
(4) When the gas is under high pressure, what is the relation between σ_o and pressure P? (4%)

7. The complex amplitude of the electric field of a monochromatic electromagnetic wave of wavelength λ_0 traveling in free space is $\mathbf{E(r)} = E_0 \sin \beta y \exp(-j\beta z)\hat{y}$. (a) Determine a relation between β and λ_0 (b) Derive an expression for the magnetic field vector **H(r)**. (c) Determine the direction of flow of optical power. (d) This wave may be regarded as the sum of two TEM plane waves. Determine their directions of propagation. (10%)

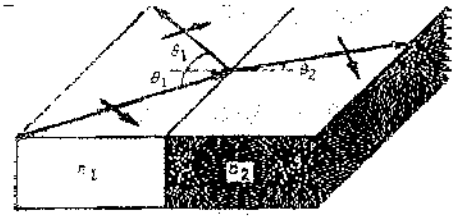
8. A pulse of initial width τ_0 is transmitted through a graded-index fiber of length L kilometers and power-law refractive-index profile with profile index p. The peak refractive index n_1 is wavelength-dependent with $D_\lambda = -(\lambda_0 / c_0)\, d^2 n_1 / d\lambda_0^2$, Δ is approximately independent of wavelength, σ_λ is the source's spectral width, and λ_0 is the operating wavelength. Discuss the effect of increasing each of the following parameters on the width of the received pulse: L, τ_0, p, $|D_\lambda|$, σ_λ, and λ_0 . (10%)

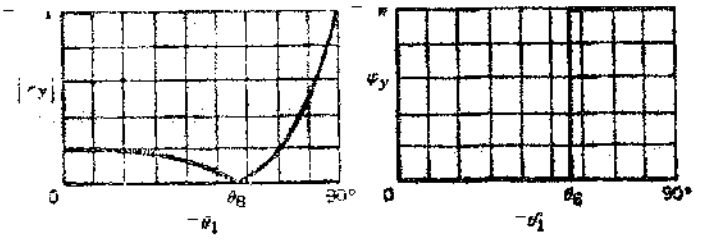
$$n^2(r) = n_1^2\left[1 - 2\left(\frac{r}{a}\right)^p \Delta\right], \qquad r \le a$$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1},$$

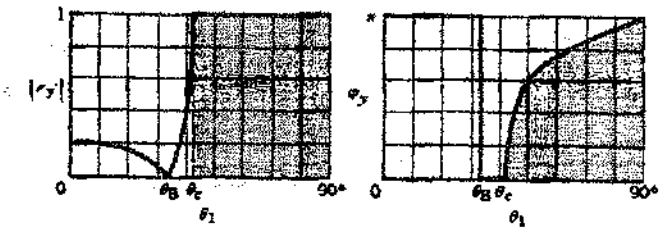


9. The following plots, extracted from the textbook Fundamentals of Photonics (by Saleh and Teich), show a TM polarized plane wave incident on a plane interface from a first dielectric of refractive index n_1 to a second dielectric of refractive index n_2 with an incident angle θ_1 and a refracted angle θ_2. The reflection coefficient is denoted by r_y and the reflection phase is denoted by φ_y, and the Brewster angle is denoted by θ_B.





External Reflection $n_2 > n_1$ for TM or p-polarization



Internal Reflection $n_1 > n_2$ for TM or p-polarization

(1) Give a physical explanation that there exists a Brewster angle at which no TM light is reflected at the interface? (5%)

(2) Explain why the reflection phase is reversed by 180° at the Brewster angle? (5%)

(3) If the incident wave is randomly polarized, explain why the reflected light and the refracted light form a right angle at an incident angle equal to the Brewster angle. (5%)