國 立 清 華 大 學 命 題 紙

一百零二 學年度第二學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號 共 頁第 頁 ＊請在試卷(答案卷)內作答

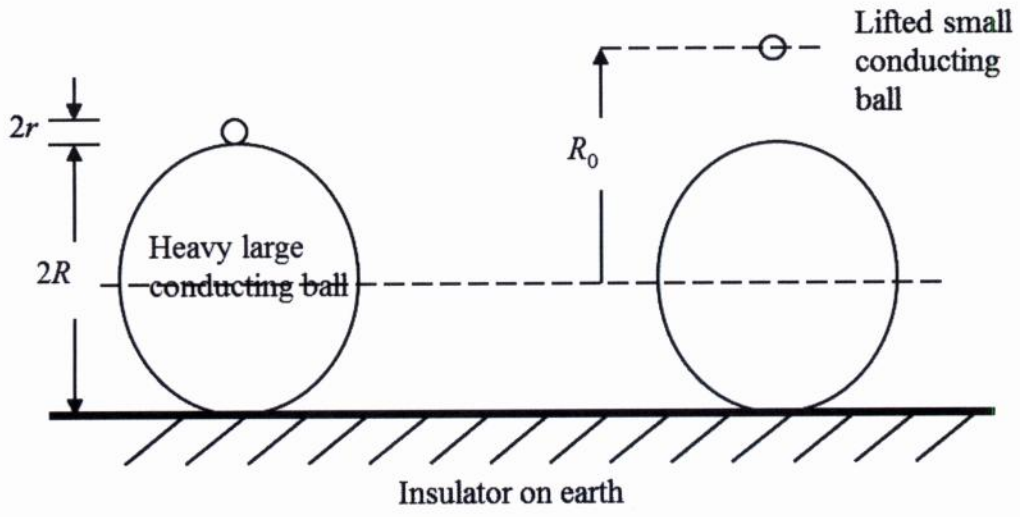
1. (15%) On an insulated earth surface, a small conducting ball of radius r is placed atop a large, heavy conducting ball of radius R, where $R >> r$. The two balls touch each other. An amount of charge Q is deposited onto the large conducting ball and distributed over both ball surfaces. Apparently for $R >> r$, nearly all the charges are on the surface of the large conducting ball. Owing to the repellent Coulomb force from same charges, the small ball is lifted to a height of $R_0 >> R + r$ from the center of the large ball. The small ball has a mass m and the earth has an acceleration constant g.

   (a) (5%) What is the charge deposited on the small ball?

   (b) (5%) What is the ratio of the surface potential of the small ball to that of the large ball?

   (c) (5%) Suppose when the small ball just leaves the surface of the large ball, both ball surfaces have the same electric potential. What is the ratio of the surface electric field of the small ball to that of the large ball?

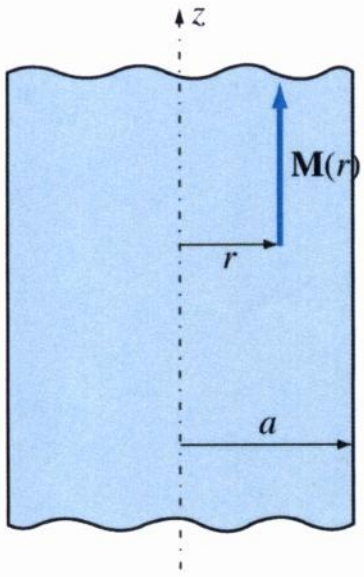
   State all the assumptions to reach your answers.



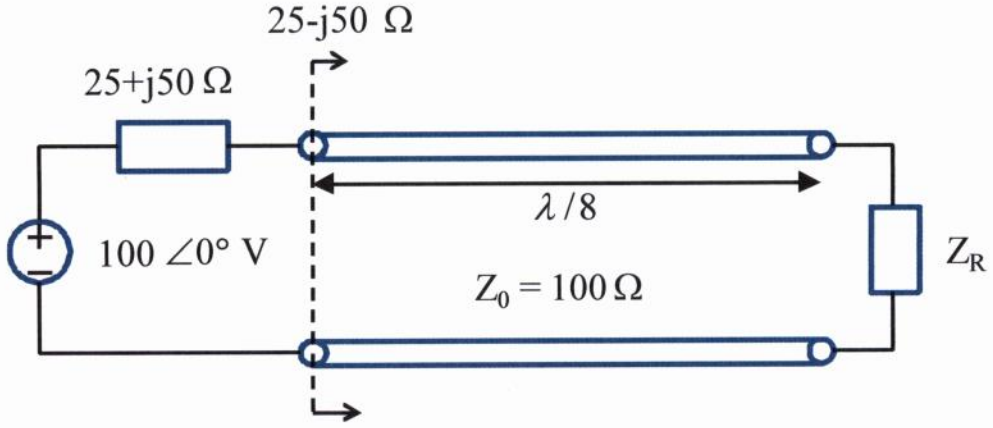
2. (a) (3%) What is meant by a cutoff frequency of a "waveguide"?

   (b) (7%) Sketch the electric and magnetic field lines for (i) the TM_1 mode and (ii) the TE_2 mode in a rectangular waveguide.

3. (10%) You have an infinitely long cylinder of radius a positioned in air. The cylinder has a magnetization of $\vec{M}(r) = \hat{a}_z M_0 (1 - r^2/a^2)$. Find the distribution of magnetization currents.



4. (10%) Consider a lossless transmission line as shown in the following figure. If the input impedance of the line is (25-j50)Ω, please answer the following questions.



   (a) (5%) the value of the load impedance Z_R

   (b) (5%) the power transferring to the load Z_R

5. (10%) Consider a symmetric dielectric slab waveguide, in which the index of the core is 2.35 and that of the classing is 2.25. The thickness of the core layer is 2.5 um.

   (a) (5%) Please evaluate the number of modes (including both TE and TM) guided by this waveguide at 1300 nm.

   (b) (5%) What is the range of wavelength for this waveguide to be single-mode?

6. (15%) Consider the boundary conditions between a dielectric and a perfect metal:

   (a) (5%) Can a dc magnetic field injected from the dielectric side penetrating into the perfect metal?

   (b) (5%) Can an ac magnetic field injected from the dielectric side penetrating into the perfect metal?

   (5%) Please explain.

7. (15%) Consider a coaxial cable with non-cylindrically symmetric inner and outer conductors biased at dc voltages of V_0 and 0, respectively (Figure 1). The space between the two conductors is vacuum.

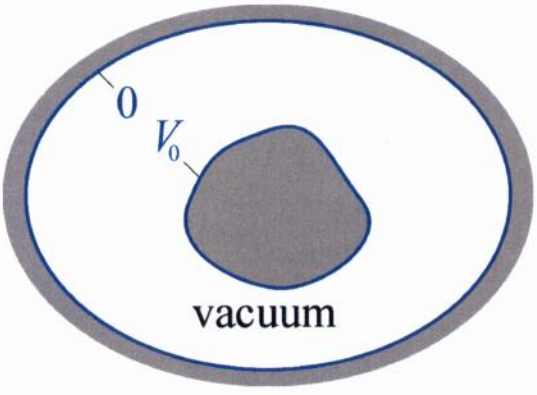


Figure 1

   (a) (10%) How to calculate the electric field distribution in the vacuum region? Just describe the method without actual calculation.

   (b) (5%) Prove the Laplace's equation by the fundamental postulate $\nabla \cdot \vec{D} = \rho$, constituent relation $\vec{D} = \varepsilon \vec{E}$, definition of potential $\vec{E} = -\nabla V$, and definition of scalar Laplacian operator $\nabla^2 V \equiv \nabla \cdot (\nabla V)$.

8. (15 %) When there is a relative motion between a time-harmonic source and a receiver, the frequency of the wave detected by the receiver tends to be different from that emitted by the source. This phenomenon is known as the Doppler effect. Let us assume that a light transmitter of a time-harmonic wave of a frequency f moves with a velocity u (assume $u << c$) at an angle θ relative to the direct line to a stationary receiver.

(a) (5%) Please derive and show that the frequency of the received wave is $f' = \dfrac{f}{1 - \dfrac{u}{c}\cos\theta}$.

(b) (5%) If the transmitted signal has a spectral linewidth of $\Delta\upsilon$, what would be the linewidth of the received signal after the Doppler effect?

(c) (5%) How would the linewidth of the received signal change if the target has a rough surface?