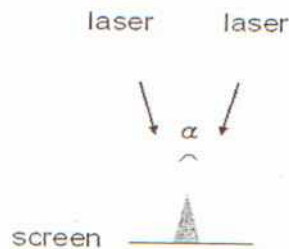


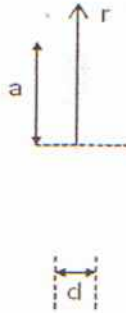
1. Graded-Index (GRIN) lens is made from a material showing radial refractive index distribution as $n(r) = n_0(1 - \alpha r^2)$, where r is the radial distance from the optical axis, α is some design parameter. Now answer the following questions:
- a. (6%) Draw a schematic figure and explain how a 0.25-pitch GRIN lens can be used to construct an optical fiber collimator?
 - b. (4%) Discuss whether other pitch lengths can be used as fiber collimators? What are the pros and cons?

2. (4%) Does the standing wave $\vec{E} = \vec{E}_0 \sin(kz) \cos(\omega t)$ satisfy the wave equation? If yes, please show your calculation. If no, please explain your reason.

3. (6%) A laser beam emitting red light of 632nm is used with a beamsplitter to produce two coherent beams. Both are reflected from plane mirrors and brought together at the screen with an angle $\alpha=1^\circ$ as depicted in the figure (the screen normal bisects the angle). Please show your derivation and find the period of the interference fringe observed on the screen.



4. (5%) A lens is made from a glass of constant thickness where there is an index of refraction variation in the radial direction $n(r)$ and $n(0) = n_0$. Given a disk of radius a and thickness d as shown below, please find the radial variation of the index of refraction $n(r)$ which will produce the equivalent of a conventional lens with a focal length f . You may assume $f \gg r$ and a thin lens ($d \ll a$).



5. Consider a Gaussian beam with Rayleigh range z_0 transmitting through an optical device whose ray-matrix is $[ABCD]$. If the beam waist is at a distance z from the front surface of the device, find the beam parameter q at the output surface. (10%)

6. (15%) Write down the transfer function of a coherent light wave propagating under water. Write down the Fresnel approximation of the transfer function. A laser beam ($\lambda=500$ nm) emitting from submarine located 300 m below the ocean surface propagates through a 10×10 (cm) window. The laser beam has a Gaussian profile with the waist of 5 cm at the window. What's the laser intensity profile on the water surface assuming the ocean is completely calm?

7. (15%) Electromagnetic optics.
- (a) What are the differences among ray optics, wave optics, and electromagnetic optics? (5 points)
 - (b) What is the meaning of dispersion? Why are nearly all the materials dispersive? (5 points)
 - (c) What happens when a short pulse passes through a dispersive medium? Justify your answer by simple mathematical modeling.

8. (10%) Polarization and crystal optics:

(a) If the Jones vector of a monochromatic plane wave is:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

Plot the trajectory and denote the sense of rotation of the corresponding E-field. (5 points)

(b) What happens when this monochromatic plane wave passes through a quarter-wave retarder? Justify your answers. (5 points)

9. (a) Give the physical meanings of photon lifetime and Q factor of an optical

resonator. Explain how and why they are related. (7%)

(b) Give two applications for the Fabry-Perot resonator. Explain in details how they work. (8%)

10. (10%) Consider the following three basic hypotheses:

(a) For sufficiently small Δt , the probability of a single impulse occurring in the time interval t to $t + \Delta t$ is equal to the product of Δt and a real nonnegative function $\lambda(t)$; thus $P(1; t, t + \Delta t) = \lambda(t)\Delta t$.

(b) For sufficiently small Δt , the probability that more than one impulse occurs in Δt is negligibly small (i.e. there are no "multiple" events); hence $P(0; t, t + \Delta t) = 1 - \lambda(t)\Delta t$.

(c) The numbers of impulses in nonoverlapping time intervals are statistically independent.

Then by using the three fundamental hypotheses above show that the photocount statistics for light from a single-mode, amplitude-stabilized laser radiations obeys the *Poisson process*.