國 立 清 華 大 學 命 題 紙

九十八學年度第一學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號______ 共____ 頁第___ 頁 ＊請在試卷(答案卷)內作答

1. (Total 9%) In a given region of space, the vector magnetic potential is given by:

$$\vec{A} = \hat{a}_x 5(\cos \pi y) + \hat{a}_z (2 + \sin \pi x) \quad \text{(Wb/m).}$$

1a. (4%) Please determine $\vec{B}$.

1b. (5%) Please calculate the magnetic flux passing through a square loop with 0.25-m-long edges if the loop is in the x-y plane, its center is at the origin, and its edges are parallel to the x- and y-axes.

2. (Total 8%) The interface between two regions is shown in Fig. 1. The current density on the surface is 0. Region 1 is air, and Region 2 is a magnetic material with $\mu=3.1\mu_0$. The magnetic field in Region 2 is $\vec{H}_2 = 4\hat{a}_x + 3y^2\hat{a}_y + 5\hat{a}_z$. Determine the magnetic flux density $\vec{B}_1$ in Region 1.

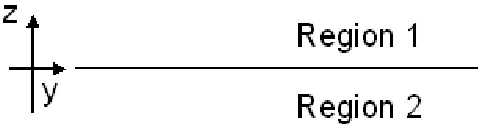

Fig. 1

3. (Total 8%) Two infinite parallel planes (parallel to y-z plane) carry equal but opposite uniform charge densities ±σ as shown in Fig. 2. Please find the field (<u>including magnitude and direction</u>) in each of the three regions: (i) to the left of both, (ii) between them, (iii) to the right of both. Please show your work. Answers without explanation won't count.

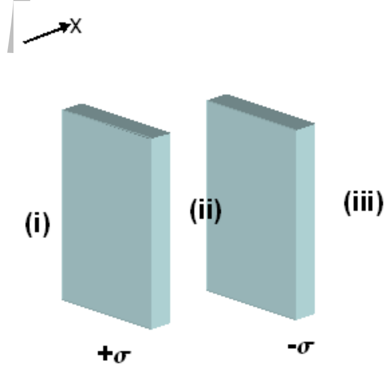

Fig. 2

4. (Total 16%) For a propagating wave, each component k_j (j = x, y, z) takes the values satisfying $-|k|^2 \leqq k_j \leqq |k|^2$, where $k^2 = k_x^2+k_y^2+k_z^2 = (n\omega/c)^2$. When a TE wave is totally reflected from a dielectric interface, the field on side *1* forms a perfect standing wave (Fig. 3). The maximum of the wave has phase φ (shown as \phi in Fig. 3) with respect to the interface; $\Theta_G = 90^o - \varphi$, which is the Goos-Hanchen shift.

4a. Solve the boundary value problem in the case when $\Theta_1 > \Theta_c$ and show that $|E^+| = |E^-|$.

4b. Develop an expression for $\mathrm{Cot}\Theta_G$ in terms of Θ_1 and Θ_c (Θ critical).

4c. From the uncertainty relation of Fourier transform, show that the minimum variation of real space (Δr) equals to $\lambda_0/(4\ n)$, which defines the resolution limit.

4d. Show that if one wants to have *sub-wavelength resolution*, one possibility is to use total internal reflection.

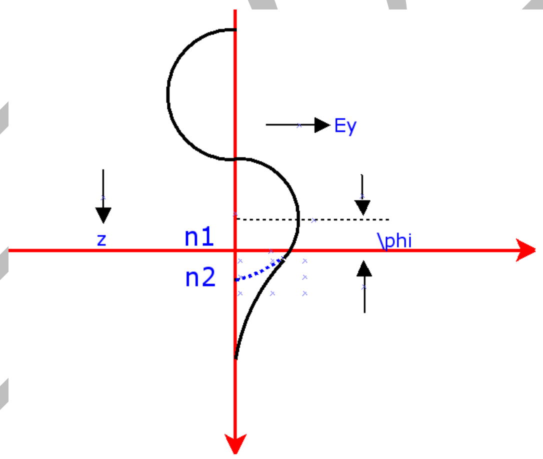

Fig. 3

5. (Total 14%) With the Maxwell's equations listed below, and by introducing the vector and scalar potentials, $\mu_0 H = \nabla \times A$ and $E = -\partial A/\partial t - \nabla \Phi$, respectively, derive the wave equations for A and Φ with the *Lorentz gauge*.

$$\nabla \times E = -\frac{\partial}{\partial t}\mu_0 H - \frac{\partial}{\partial t}\mu_0 M, \qquad \nabla \cdot \epsilon_0 E = -\nabla \cdot P + \rho,$$
$$\nabla \times H = \frac{\partial}{\partial t}\epsilon_0 E + \frac{\partial}{\partial t}P + J, \qquad \nabla \cdot \mu_0 H = -\nabla \cdot \mu_0 M.$$

6. (Total 10%) Taipower distributes electric power via 60-Hz sinusoidal waves traveling in the air. If a source voltage $V_{AA'}(t) = V_0 \cos(2\pi \cdot 60 \cdot t)$ is generated by a power plant in southern Taiwan, what is the voltage $V_{BB'}(t)$ received by a user in northern Taiwan distanced from the power plant by 300 km? By your result, whether the "lumped circuit" model is appropriate to analyze the power transmission over 300 km? Justify your answer.

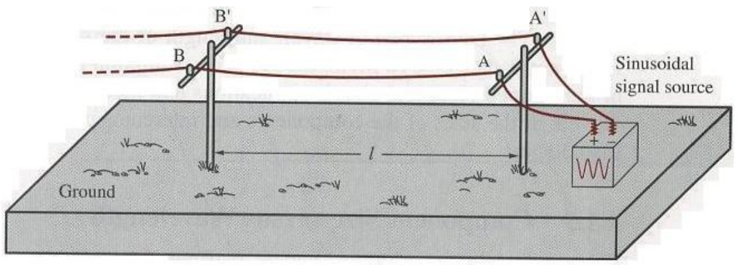

Fig. 4

7. (Total 15%)

7a. (10%) An ideal parallel-plate transmission line (Fig. 5) can be modeled by distributed equivalent circuit (Fig. 6). Please briefly explain why an infinitesimal transmission line of length Δz can be modeled by a "series" inductance $L\Delta z$ and a "shunt" capacitance $C\Delta z$?

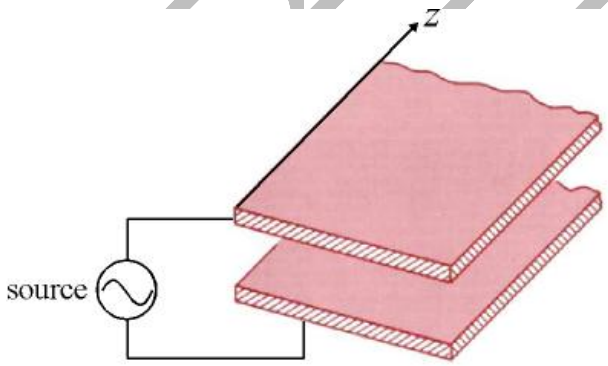

Fig. 5

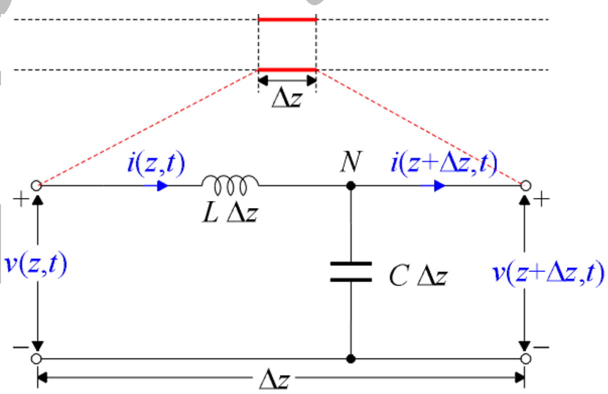

Fig. 6

7b. (5%) By Fig. 6, and Kirchhoff's voltage and current laws, derive a second-order partial differential equation for voltage signal $v(z,t)$.

8. (Total 20%)

Consider the parallel-plate waveguide of two perfectly conducting plates separated by a dielectric medium with ε and μ, as shown below.
Start with the wave equation

$(\frac{d^2}{dx^2}+h^2)\ E_z = 0$, where E_z is the z component of a TM mode of the waveguide and h^2 is given as $\gamma^2 + k^2$ with k^2 equal to $\omega^2\mu\varepsilon$. Note that the wave has been expressed as $E_z\ e^{-\gamma z}$.

8a. (10%) Find E_z for TM modes.

8b. (10%) Then find H_x and E_y.