國 立 清 華 大 學 命 題 紙

一百學年度第一學期 光電工程研究所 博士班研究生資格考試
科目 電磁理論 共____頁第____頁 ＊請在試卷(答案卷)內作答

1. Consider a parallel-plate capacitor filled with air. The bottom and top conducting plates of area $S = w \times L$ are uniformly deposited with free charges $\pm Q$, respectively (Fig. 1a). Assume $w, L >> d$, and the fringing effect at the edges ($x = \pm w/2$) is negligible.

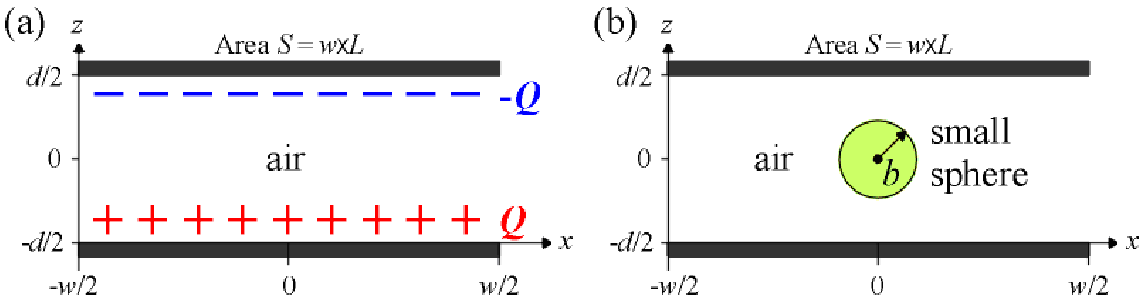
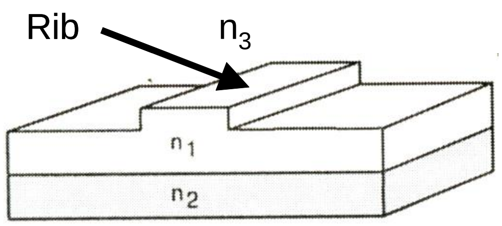


Fig. 1

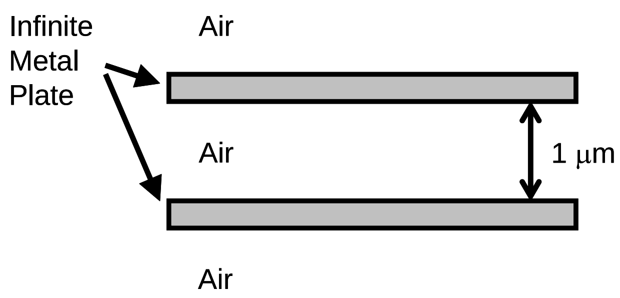
1a) (15%) What are the electric flux density $\vec{D}$, electric field intensity $\vec{E}$, and voltage difference $V_a \equiv V(-d/2) - V(d/2)$ between the conducting plates? What is the corresponding capacitance C_a? (*Hint*: Use Gauss's law or the boundary condition $D_{1n} - D_{2n} = \rho_s$. Capacitance is defined as $C \equiv Q/V$.)

1b) (10%) Place a small <u>conducting</u> sphere (with radius $b << d$) around $(x, z) = (0,0)$ (Fig. 1b). Are the voltage difference $V_c \equiv V(-d/2) - V(d/2)$ and the capacitance C_c in the presence of a conducting sphere <u>larger or smaller</u> than V_a and C_a calculated in Problem 1a, respectively? Justify your answer.

2. (15 %) Please give an explanation why the guided wave is confined in the rib region, as shown in the following figure. Consider the refractive indices n1 > n2 > n3.



3. (10 %) A metallic planar waveguide is shown as follow. What is the frequency range so that only one TE guided mode exists? Suppose the metal is a perfect conductor.



4. (30%) The electric field intensity of a harmonic electromagnetic wave in vacuum is given by $\vec{E} = \hat{x}E_0 \cos(\omega t - kz + \phi)$, where $\hat{x}$ is a unit vector along the x direction, E_0 is a constant, t is a temporal variable, z is spatial variable, and ϕ is the starting phase.

(1) Prove that the wavelength of this wave is $2\pi/k$. (2%)

(2) Prove that the frequency of this wave is $2\pi/\omega$. (2%)

(3) Prove that the phase velocity of this wave is ω/k. (2%)

(4) Prove that the wavefront of this wave is a plane. (3%)

(5) Derive from Maxwell's equations the expression of the magnetic field intensity of this wave. (3%)

(6) Suppose the region $z \geq 0$ is filled with a perfect conductor.

   a. What is the expression of the reflected electric field intensity in the region $z \leq 0$? (3%)

   b. What is the expression of the reflected magnetic field intensity in the region $z \leq 0$? (3%)

c. What is the <u>total</u> time-averaged Poynting vector in the region $z \leq 0$? A quick answer from physical arguments is acceptable. (3%)

(7) Suppose the region $z \geq 0$ is filled with a lossless, nonmagnetic dielectric with a relative permittivity of $\varepsilon_r > 1$.

a. What is the expression of the reflected electric field intensity in the region $z \leq 0$? (3%)

b. What is the expression of the reflected magnetic field intensity in the region $z \leq 0$? (3%)

c. What is the <u>total</u> time-averaged Poynting vector in the region $z \leq 0$? A quick answer from physical arguments is acceptable. (3%)

5. (10%) (a) Do the boundary conditions derived for electrostatics and magnetostatics remain valid for time-varying fields? Please explain your reasoning. (b) Please use Maxwell's equations to derive the boundary conditions of $\vec{B}$ for an interface between two media with permittivity μ_1 and μ_2. (No credit is given to a correct answer without any explanation or derivation.)

6. (10%) (a) What is a transmission line? When should transmission line effects be considered? (b) For a lossless transmission line, λ=20.7 cm at 1 GHz, please find the relative permittivity ε_r of the insulating material.