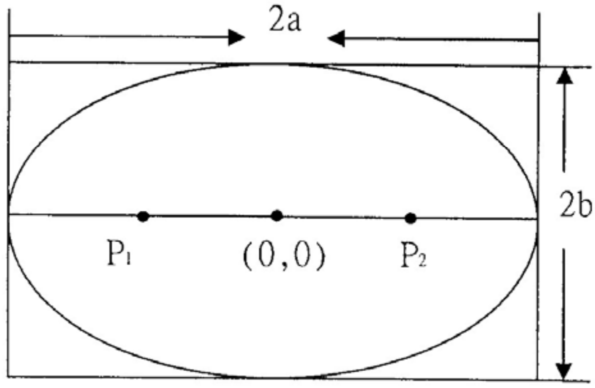
國立清華大學命題紙

九十四 學年度第一學期 光電工程研究所 博士班研究生資格考試

科目 光電子學 科號______ 共___頁第___頁 ＊請在試卷(答案卷)內作答

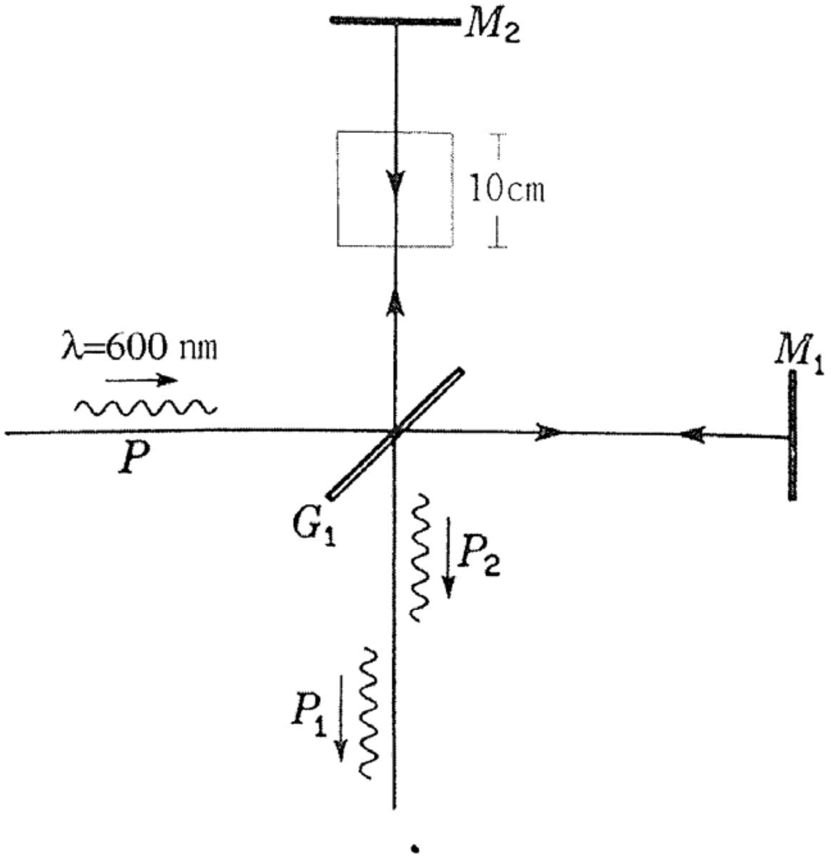
1. Show that the elliptical mirror below will focus all the rays from one of its foci, e.g., $P_1(-\sqrt{a^2-b^2},0)$, onto the other focus, $P_2(\sqrt{a^2-b^2},0)$. (10%) Explain it by the Fermat's principle. (5%)



2. A laser emits three adjacent longitudinal modes at wavelength $\lambda = 1$ μm with mode spacing of 500 MHz. The linewidth of each mode is 10 MHz. We want to resolve these modes using an air-spaced Fabry-Perot.
   (1) What is the requirement on the cavity length L of the Fabry-Perot? (5%)
   (2) When the value of L is given, what is the requirement on the finesse F of the Fabry-Perot to well explore the spectral profile of each laser mode? (5%)
   (3) Find the minimum variation ΔL_{min} needed to scan the Fabry-Perot across a laser mode spacing. (5%)

3. The transition between two energy levels has a spontaneous emission wavelength $\lambda = 1$ μm and life time $t_{sp} = 10^{-8}$ sec. Find the transition cross section $\sigma(\nu_0)$ at its line center, assuming that the line broadening is homogeneous with linewidth $\Delta\nu = 10/t_{sp}$. (5%)

4. Suppose we place a chamber 10 cm long with flat parallel windows in one arm of a Michelson Interferometer that is being illuminated by 600-nm light. If the refractive index of air is 1.0029 and all the air is pumped out of the cell, how many fringe-pairs will shift by in the process (after pumping)? (7%)



5. A Gaussian beam is transmitted through a thin lens of focal length f.
(a) Show that the locations of the waists of the incident and transmitted beams, z and z′, are related by

$$\frac{z'}{f} - 1 = \frac{z/f - 1}{(z/f - 1)^2 + (z_0/f)^2} \qquad (4\%)$$

(b) The beam is collimated by making the location of the new waist z′ as distant as possible from the lens. This is achieved by using the smallest ratio z_0/f (short depth of focus and long focal length). For a given ratio z_0/f, show that the optimal value of z for collimation is $z = f + z_0$. (6%)

(c) If $\lambda = 1\ \mu m$, $z_0 = 1$ cm and f = 50 cm, determine the optimal value of z for collimation, and the corresponding magnification M, distance z′, and express the width W_0' of the collimated beam with the incident beam waist W_0. (4%)

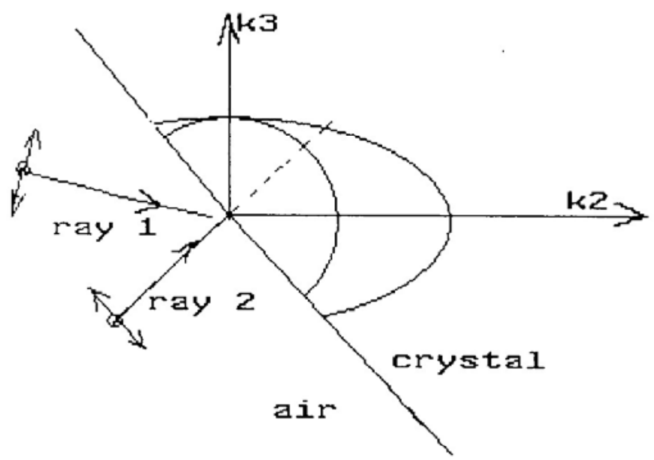
6. An electromagnetic wave in free space has an electric field $E = f(t - z/c_0)\hat{y}$, where $\hat{y}$ is a unit vector in the y direction, $f(t) = \exp(-t^2/\tau^2)\exp(j2\pi\nu_0 t)$, and τ is a constant. Describe the physical nature of this wave and determine an expression for the magnetic field vector. (8%)

7. The light intensity pattern shown on an observation plane in Young's double-pinhole experiment would be clear fringes if a monochromatic light is used to light the pinholes. What would happen to the fringes if the light source is not monochromatic, but a partially coherent one ? (3%) Explain why. (7%)

8. Suppose you have a uniaxial crystal and an incident optical ray carrying two polarizations. The intersections of the two k surfaces (i.e., normal surfaces) with the y-z plane of the crystal are shown to be a part of an ellipse and a circle in the figure below.

(a) Draw two vectors to show how the incident ray would propagate in the crystal if the ray is incident normally (i.e., follows the direction of ray 2). (7%)

(b) What happens if the incident ray follows the direction of ray 1 ? (7%)



9. Show that the *power spectral density*, $S(\nu)$, defined as $S(\nu) \equiv \lim_{T\to\infty} \frac{1}{T} < |V_T(\nu)|^2 >$, is the Fourier transform of the autocorrelation function, $G(\tau)$, i.e.

$$S(\nu) = \int_{-\infty}^{\infty} G(\tau)\exp(-j2\pi\nu\tau)\ d\tau\ ,$$

where $V_T(\nu) = \int_{-T/2}^{T/2} U(t)\exp(-j2\pi\nu t)\ dt$, and $G(\tau) = \lim_{t\to\infty} \frac{1}{2T} \int_{-T}^{T} U^*(t)\ U(t+\tau)\ dt$. (5%)

10. The *mutual intensity* of an optical wave at points on the x axis is given by

$$G(x_1, x_2) = I_0 \exp[-\frac{(x_1^2 + x_2^2)}{W_0^2}]\exp[-\frac{(x_1 - x_2)^2}{\rho_c^2}]\ ,$$

where I_0, W_0, and ρ_C are constants. Derive an expression for the normalized mutual intensity $g(x_1,x_2)$ and sketch it as a function of $x_1 - x_2$. What is the physical meaning of the parameters I_0, W_0, and ρ_C? (7%)