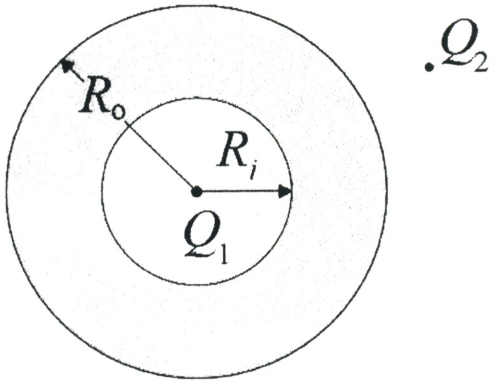
1. (10%) We live inside of a huge capacitor. The earth's surface and the electrosphere that is 25 km above have a potential difference of 300 KV. The radius of earth is 6370 km. The permittivity in the air is about $8.85\mathrm{x}10^{-12}$ pF/m. Estimate the capacitance of earth and the energy stored in the system.

2. (10%) Consider a charge Q_1 located at the center of a spherical conducting shell of inner and outer radii R_i and R_o, respectively.



(a) What is the E-field outside the shell ($R>R_o$)?

(b) What should we do to isolate the influence of Q_1 from another charge Q_2 outside the shell? Justify your answer?

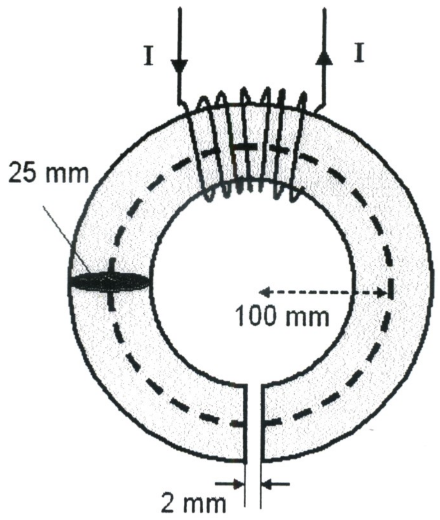
3. Analyze the working principle of a dc-motor in detail. (10%)

4. (10%) A toroidal iron core of relative permeability 3000 has a mean radius R =100mm and a circular cross section with radius b=25mm. An air gap l_g=2mm exists, and a current I flows in a 600-turn winding to produce a magnetic flux of 10^{-5} Wb. Neglecting flux leakage and using the mean path length, find

(a) the reluctances of the air gap and of the iron core

(b) $\mathbf{B}_g$ and $\mathbf{H}_g$ in the air gap, and $\mathbf{B}_c$ and $\mathbf{H}_c$ in the iron core

(c) the required current I.



5. (10%) Assume that the space between the inner and outer conductors of a long coaxial cylindrical structure is filled with an electron cloud having a volume density of charge $\rho = A/r$ for $a < r < b$, where a and b are, the radii of the inner and outer conductors, respectively. The inner conductor is maintained at a potential V_0, and the outer conductor is grounded. Determine the potential distribution in the region $\mathbf{a} < r < b$ by solving the Poisson's equation.

6. (10%) You can choose to elaborate (a) or answer all three questions in this problem to receive the full 10% credit. Grading is primarily based on your ability in reasoning a problem from known theories and/or from creative thinking.
   (a) A wire loop is dropped from a high altitude toward the center of the earth. Would the earth's magnetic field affect the motion, speed, or spinning of the wire loop, if the earth's magnetic flux intensity varies with altitude? Make necessary assumptions to justify your argument.
   (b) Explain why a time-varying magnetic field can't exist in an ideal conductor?
   (c) What assumptions are made to derive the homogeneous vector wave equation $\nabla^2 \vec{E} - \frac{1}{u^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$ from the Maxwell equations, where E is the electric field intensity, t is the time variable, and u is the moving speed of the field?

7. (10%) Consider two plane waves in free space with frequency $\omega_1 = \omega_2 = \omega$, propagation vectors $\vec{k}_1$, $\vec{k}_2$ and phases ϕ_1, ϕ_2, respectively.
   (a) Write the functional expressions for the E-field of them. (2 %)
   (b) If $\vec{k}_1 \, // \, \vec{k}_2$, can you observe the interference fringes? Why? (3%)
   (c) If $\vec{k}_1 = -\vec{k}_2$ and the amplitudes are $E_{10} = E_{20} = E_0$, find the expression for the space variation of $|E|^2$ (5%)

8. (10 %) A uniform plane wave is incident normally from a lossless medium 1 onto another lossless medium 2 through a plane boundary z = 0 in z-direction. The intrinsic impedances of medium 1 and 2 are η_1, η_2, respectively. The incident electric and magnetic filed intensity phasors are

$$E_i(z) = a_x E_{i0} e^{-j\beta_1 z}, H_i(z) = a_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

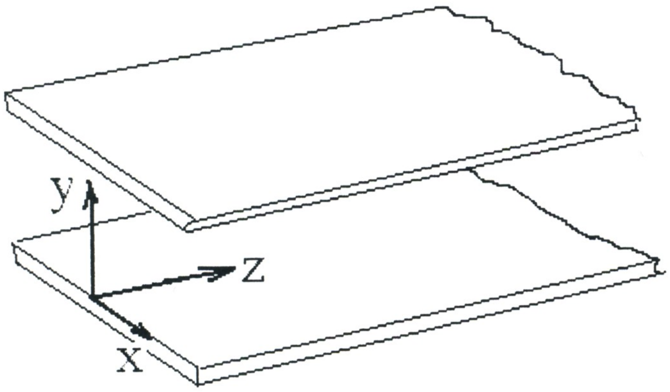
(a) By finding the electric filed intensity and magnetic field intensity of the reflected and transmitted waves, derive the expressions of the reflection coefficient Γ and transmission coefficient τ in terms of the intrinsic impedances. (show your work) (3 %)

(b) Find the expressions of the electric field intensity and magnetic field intensity in media 1 and 2. (3 %)

(c) Derive the time-average Poynting vectors in media 1 (P_1) and 2 (P_2). (2 %)

(d) From P_1 and P_2, what is the relation between the reflection coefficient Γ and the transmission coefficient τ? (2 %)

9. (10%) Consider the parallel-plate waveguide of two perfectly conducting plates separated by a dielectric medium with permittivity ε and permeability μ, as shown below. Suppose the electric field polarized in the z direction of the TM wave (designated as TM_n mode, n= 1,2 3,....) of this waveguide can be written as $E_{zn}(y, z) = E_{on}(y) \exp(-j \beta_n z)$, where β_n is a real number.



(a) Write the wave equation governing the propagation of the field $E_{zn}(y, z)$. (3%)

(b) Find $E_{zn}(y, z)$, and the magnetic field H (y, z). (7%)

10. (10%) A symmetric planar waveguide has a core thickness of 2 mm. Ignoring the dispersion of the waveguide material, we find the indices to be n1(core) = 1.50 and n2(cladding) = 1.46.

(a) Is the waveguide single-mode or multimoded at l = 1.5 and 1.3 mm? (5%)

(b) What is the range of wavelength in which this waveguide is single-mode? (5%)