1.

(a) Write down the Maxwell's equation that implies the non-existence of magnetic charges and briefly discuss it. (6 points)

(b) However there is an analogy to an electric dipole. Write down one expression for magnetic dipole moment and explain it. (6 points)

(c) With the definition of the volume density of magnetic dipole moment, M, we can express a magnetized material in terms of a volume current density $J_m = \nabla \times M$ and a surface current density $J_{ms} = M \times a_n$. Draw a figure to demonstrate the relationship between J and M. (9 points)

2.

Two parallel wires of radius a are shown below, separated by a distance b. When a voltage V is applied between the wires, a charge density of $\pm \rho_l$ is induced on each wire. When b>>a, it can be shown that

$$\rho_l \approx \pi \varepsilon V \ln(b/a) \qquad C/m$$
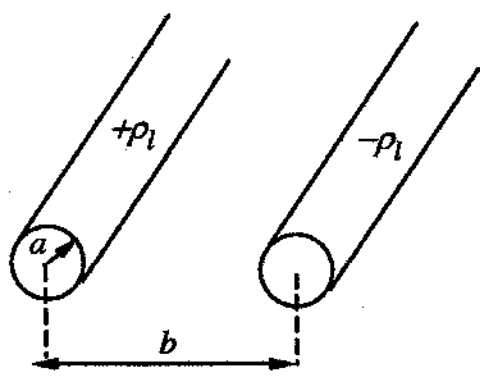
a) First, find an expression for the capacitance per unit length for this structure. (4 points)

b) Find the electrostatic energy per unit length. (4 points)

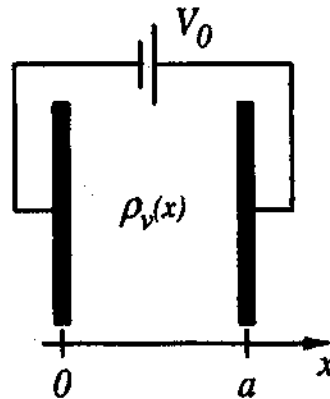Find the electrostatic force of attraction per unit length. (5 points)

3.

A charge distribution $\rho_v(x)$ fills the region between two parallel plates as shown below. The plates are separated by a distance a, and the electrostatic potential between the plates is found to be

$$\Phi(x) = V_0 \sin\left(\frac{\pi x}{2a}\right)$$

a) Find the electric field vector $\bar{E}$ between the plates. (4 points)
b) Find the charge density $\rho_v(x)$ between the plates. (5 points)
c) Find the total charge per unit area between the plates when a = 1 m,
　$\varepsilon_r = 1$, and $V_0 = 1$ Volt. (5 points)

4.

For the following three problems, demonstrating your thinking path is more important than working out the mathematic details. Never give up even thought you forget all the formulas.

(a)(12 points) Prove that in a good conductor the phase angle between the electric and magnetic fields of a plane wave is different by 45°.

(b) (12 points) A plane wave is incident on a plane interface between a first dielectric and a second dielectric. The first dielectric constant ε_1 is larger than the second one ε_2. Prove that no electromagnetic power flows into the second dielectric if the incident angle is larger than the critical angle $\theta_c > \sin^{-1}(\sqrt{\varepsilon_2/\varepsilon_1})$. The incident angle is defined to be the angle between the surface normal of the interface and the propagation direction of the incident plane wave.

(c) (12 points) A randomly polarized plane wave is incident on a plane interface between two dielectrics at the Brewster angle. Prove that the propagation directions of the reflected and refracted waves form a right angle.

5.

(16 points)Consider a wave propagating along a waveguide formed by two interfaces between a guiding layer (with a refractive index of N_1) and the surrounding media both with a refractive index of N_2. Note that the wave propagation can be considered as a ray totally-internal-reflected at the two interfaces. Suppose the guided wave has a propagation constant of β in the z axis direction. Explain why β falls in the range

$$k_0 N_2 < \beta < k_0 N_1 .$$